

DOCUMENT RESUME

ED 162 875

SE 025 375

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TITLE Studies in Mathematics, Volume XVIII: Puzzle Problems and Games Project. Final Report.
INSTITUTION Stanford Univ., Calif. School Mathematics Study Group.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 68
NOTE 218p.; For related documents, see SE 025 371-374 and ED 143 544-557; Not available in hard copy due to marginal legibility of original document
EDRS PRICE MF-\$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS Curriculum; *Discovery Learning; *Elementary School Mathematics; Elementary Secondary Education; *Games; Instruction; *Instructional Materials; Mathematics Education; *Secondary School Mathematics; *Teaching Guides
IDENTIFIERS School Mathematics Study Group

ABSTRACT

This is a self-contained manual for use by teachers in preparation for classroom presentations. One of the goals of the report is to show how games and puzzles can provide effective means for developing mathematical understanding and skills. The authors indicate that this type of activity is well adapted for discovery teaching techniques. The report is organized into two main parts. The first part contains experimental units that were tested in the classroom. The topics in this part include: (1) nix-type games; (2) polyominoes; (3) symmetry; (4) a counting machine; (5) finding the greatest common divisor; (6) linear function games; and (7) games with addition tables. The second part consists of the report of a project to compile a list of games and puzzles appropriate for use in the mathematics classroom. Twenty-seven papers contain (in addition to the above list) activities such as: (1) magic squares; (2) Fibonacci problems; (3) geometric puzzles; (4) numerical oddities; and (5) powers and primes. (MF)

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Studies in Mathematics

VOLUME XVIII

Puzzle Problems and Games Project
Final Report

SE 025 375



Studies in Mathematics

Volume XVIII

Puzzle Problems and Games Project

Final Report

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Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

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Figure 1. The effect of the number of trials on the number of correct responses. The number of correct responses was significantly higher for the 10 trials condition than for the 5 trials condition.

The major aim of the project was to develop a national potential in
 the field of mathematics in schools. It was also for classroom use. The idea must
 be presented in a way which is not only attractive to the students but
 interesting to them. The experimental nature of the game or puzzle problem
 is not lost. It is a project which requires special training session for
 the teacher, and the particular game or puzzle will be
 carefully studied. The main aim of the project was to pre-
 pare some experimental material. These games and puzzles which were self-
 contained and which could be used in special training sessions in order to re-
 late mathematics to the real world.

6

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It is pointed out that the use of the linear approximation from perturbation theory, that within a few percent, even at a polarization of 0.5, leads to the conclusion that the probability is small of asymmetry.

From a developmental perspective, this new child, working child is different from the child of the industrial device. Children do not like to be pushed into the child and will exert a great deal of mental effort in order to be able to do the difficult.

1. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Lichtenthaler and Whistler (1973).

4. The following is a list of the main steps in the computing process:

- a. The first step is to identify the problem to be solved.
- b. The second step is to design a solution.
- c. The third step is to implement the solution.
- d. The fourth step is to evaluate the solution.

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THIRD PROGRESS REPORT

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1. *Long, J. L. 1979.*
 2. *Long, J. L. 1980. J. Fish. Res. Board Can.*
 3. *Long, J. L. 1981. J. Fish. Res. Board Can.*

Mr. Joseph E. Dineen
 Manhattan Federal Reserve
 New York

Dr. Brown died on
 12th August 1901.
 Married 1872, 1 wife.

Mr. J. Edgar Hoover
 Department of Justice
 Building, Washington

On December 11, 1961, the Honorable Earl Warren of Maryland worked with Mr. Frost and I and, along with Mr. Frost, advised me of his writing project on the assassination of President Kennedy.

APPENDIX A

LIST OF STUDY AND RESEARCH INSTITUTIONS

International, European, and Swiss

Mr. J. J. J. J. J.
Institute
Institute, 1911-1912

Mr. J. J. J. J. J.
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NIM-TYPE GAMES

(Developed by J. I. Illworth and J. Hill)

INTRODUCTION

Nim-type games are played by two players and may be described as follows: A starting sequence of numbers is given; the players play alternately, each play consisting of a modification of the sequence of numbers within limits prescribed by the rules of the game. The first player to change the sequence into one of a given set of target configurations wins the game.

In fact, the simplest Nim-type game is that in which the starting sequence consists of the single number zero. Each play consists of adding one of the numbers 1, 2, ... to the number previously obtained. The first player to reach a given target number, for example 10, wins.

A slightly more complicated game of this type is a game of "11". Again the starting sequence consists of the single number zero. Each play consists of adding one of the numbers 1, 2, 3, ... to the number already obtained. The target number in this case is the number 11.

In the standard game of Nim, the starting sequence consists of the three numbers 3, 4, 5. A move consists of replacing any one of the numbers in the sequence by a smaller whole number. The first player to obtain the sequence 0, 0, 0 wins the game.

In a more complicated game of this type the following may follow: The starting sequence consists of the numbers 1, 2, 3. Each play consists of adding 1, 2, 3, ... to one or more of the numbers of the sequence. The first player to obtain a sequence containing the number 10 wins.

It should be noted that in all these games the game need not be played with rods. It is possible that the players are provided with a sequence of rods of different lengths. The first player selects a rod and places it on the playing table. The second player then selects a rod and places it on the playing table so that it extends to the rod already placed in the previous play. The players continue to play in this manner. The player who is first to place a rod so that its end coincides with a certain target location wins the game.

It should be noted that it is possible to define an wide variety of Nim-type games.

Experience has shown that games of this type are capable of stimulating the interest of persons of nearly every age group. However, we are not principally interested in the entertainment value of the game, but rather in ways in which they can contribute to mathematical skills and understanding.

Level 1 Nim-type games with a constant number of elementary addition and subtraction problems, may be generally applicable only to the early elementary grades. Increasing the number of problems and enlarging the collection sets, while complicating the arithmetic, may be more applicable to the interests of the game. Indeed, by greatly increasing the number of possible moves, it may imply, rather than make more confusing, to the players. However, the game number may still be within the level of arithmetical skill of the players.

In fact, the most significant mathematical contribution of Nim-type games is in the area of problem solving. Namely, the development of a strategy of play, based on known examples of problems in which elementary arithmetic operations are contained, and which require creative solutions. The approach is similar to that of complex problems which occur in many types of mathematics. Furthermore, the strategy for winning progresses toward solutions of more difficult problems -- the player wins more games. In fact, a similar situation exists in many problems in which the player can win every time he plays.

One of the most important mathematical problems is the problem of finding a strategy for winning a game. The problem of finding a strategy for winning a game is a problem in mathematics. The problem of finding a strategy for winning a game is a problem in mathematics. The problem of finding a strategy for winning a game is a problem in mathematics. On the other hand, the problem of finding a strategy for winning a game is a problem in mathematics. The problem of finding a strategy for winning a game is a problem in mathematics.

Nim-type games with a constant number of problems involve a little more than arithmetic. The player is required to use a strategy to win the game. The player is required to use a strategy to win the game. The player is required to use a strategy to win the game. The player is required to use a strategy to win the game.

STRATEGY FOR WINNING

Nim-type games with a constant number of problems would be similar to the game of Nim. The player is required to use a strategy to win the game. The player is required to use a strategy to win the game. The player is required to use a strategy to win the game.

adds it to the number selected by the first player. The first player then again selects 1 or 2, and adds it to the sum obtained by the second player. They continue in this manner until one of the players makes the sum equal to 7. This player is the winner.

It is important for the students to have some experience playing the game and making decisions since this is the only way they will discover that there is indeed a problem to be solved in winning the game and thus be motivated to try to find a solution. The first prerequisite in understanding a game is actual experience in playing the game. This is important for the teacher since without this experience it will be difficult to appreciate the pupils' choice of a method of play and their approach to an understanding of the game.

Since this is a fairly simple game, some pupils may be able to discover a winning strategy after playing relatively few games. These pupils should be asked not to give away the winning strategy to the other pupils. On the other hand, the discovery of a winning strategy by a few of the students may be a challenge for other pupils to do the same.

Example 2.1

When playing against a fellow classmate a winning strategy for them selves, the pupils are presented with an excellent opportunity to illustrate one of the basic principles of problem solving: If a problem does not yield to a direct approach, try a different method or change the problem and work at the solution from that angle. In this case, the teacher may suggest that the pupils try the game in which the number chosen is smaller, say 4. For the first game, the pupils should play the game using the number 4. A poor first move would be to choose 1, since this does not win. Hence, the first player should choose 2. If the second player chooses 1, the choice of the second player, the first player can choose 2 and the sum equal to 4. Having found the winning strategy, the first player can win. The pupils should now be able to discover the winning strategy for the game with the number 4. In fact, since the number 4 and the total 4, it is impossible for the first player to make the sum equal to 4, it is impossible for the second player to win. If the first player chooses 2, the second player can choose 2 and the sum equal to 4 and the first player wins. If the first player chooses 1, the second player can choose 3 and the sum equal to 4 and the first player wins. If the first player chooses 2, the second player can choose 2 and the sum equal to 4 and the first player wins. If the first player chooses 1, the second player can choose 3 and the sum equal to 4 and the first player wins.

It is important to note that the pupils are presented with a problem that is not a direct approach to the solution of the problem. This is a problem that is not a direct approach to the solution of the problem.

important that they continue until they clearly realize that the winning play consists of showing 1 on the first play and then making a run of 4 and finally 5.

(U.S. AIR FORCE) (SAC) (SAC)

For example, by trying the number 1 for a beginning, he may lose but in the play he may notice that 1 would have been a better move because he noticed a pattern in the results. In the next play he tests this to see if his conjecture about patterns works. Such a conclusion may then apply in principle to other variations. This discovery of a strategy may be entirely individual or it may be shared in a team of players.

There is probably a wide range of students who have discovered a winning strategy. It is a valuable experience to witness it work. In this way he will be making the first step in formulating a mathematical proof. The teacher's judgment must determine when this is appropriate and to whom the strategy is revealed. For example, it will be of more benefit to other students to continue playing and again to discover a strategy for themselves. The child who discovers and verifies a strategy in play may first discuss his argument with the teacher and then present the same to the class. On the other hand, a team effort in jointly conjecturing and testing may be a valuable experience. The main objective is that the achievement of a strategy of play is a product of each child's effort and not just what someone else tells him.

OTHER MODIFICATIONS

From this point on, there are several workings of modifications of the game which lead to new problems in determining the winning strategy.

INFORMATION GIVEN

The new modification is information given by setting the beginning number to be a positive whole number and, in each play, one of the numbers in the collection of numbers. The starting number in this case is zero.

OTHER DEFINITION OF

Instead of using the collection of numbers as a consecutive set of integers as in the first play, the numbers are now collected beginning with another number, for example, the set $\{1, 2, 3, 4, 5\}$. If the beginning number is zero and the target sum is 11, the winning first choice is 5 and the winning numbers for the first player are 5, 1, 4, 2, 3, 5, 1, 4, 2, 3. It should be noted that this game may not be won if the winning strategy is not followed. For example, if the sum 11 is selected, then it is impossible to get the sum 11. Also note that if the target sum is 1, then it is impossible for the first player to finish. On the other hand, the player may choose 4 or 5 in the first play. In this case, the player will discover more facts

a new game. He must also decide how best to obtain the dual values of independent discovery of strategy and the system solution of the conjectures and verification is established by a convincing argument as to why it works.

POLYOMINOES

(Prepared by E. L. Wilmer and J. Hill)

BACKGROUND MATERIAL FOR THE TEACHER

When geometry is applied in the physical sciences the relevant feature of a physical object is its shape. If an object is moved to a different position in space, its geometric properties are not altered. Hence, in the study of geometry, it is important to know whether or not a given configuration can be made to match another configuration by means of such a motion. It may even happen that a given configuration can be made to match itself under certain motions. In this case, the configuration will have certain symmetries. These symmetries, in turn, may determine important physical properties of objects having this shape.

In this unit a class of simple configurations will be introduced and the ways in which these configurations can be matched and fit together will be considered. These activities will provide a natural introduction to some of the basic ideas of geometry.

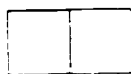
The configurations to be studied are those configurations in the plane which can be constructed by joining congruent squares along edges. Such configurations are called polyominoes. If all squares are used in constructing the configuration, it will be convenient to refer to the polyomino as an n-square.

The simplest configuration consists of a single square, i.e., the monomino or one-square. With regard to the motion of these configurations, there are two observations which should be made.

1. If two congruent squares are given in a plane, it is always possible to take one of them, turn it, and then move it across the plane in such a way that it matches exactly the second square.
2. If a square is taken a quarter turn (clockwise) about its center, then it exactly matches the square in its first position. Likewise, a half turn, three quarter turn, or full turn about the center also brings the square into a matching position.

It may also be noted that a quarter turn counter-clockwise produces the same position as a three quarter turn clockwise.

The next simplest configuration is the domino or two-square which is obtained by joining the sides of two congruent squares.



The matching of dominoes is quite similar to the matching of single squares.

1. Given two dominoes, it is always possible to take one of them, turn it, and then move it across the plane in such a way that it matches the second domino.

A domino no longer matches itself if it is turned a quarter turn about its center. The appropriate observation for dominoes is the following:

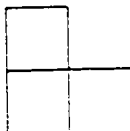
- If a domino is given a half turn or full turn about its center, it exactly matches the domino in its first position.

The plane configuration constructed by joining the edges of three squares is called a tromino or three-square.

One such configuration is the following:



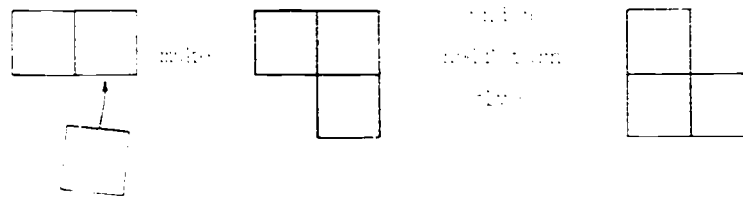
Here is still another possibility:



If it is allowed to slide a tromino on the first tromino and move it across the plane in such a way that it matches the second tromino. Thus, unlike dominoes, it is not true that every pair of trominoes can be matched by suitably turning and sliding one of them. On the other hand, this suggests the question: Can every other tromino be suitably moved so that it matches one of the two trominoes shown above?

An answer to this question requires an examination of all of the possible three-square configurations. First, clearly, that any tromino can be constructed by adjoining a square to a domino. If the edge of the square matches an

edge of a square in the domino. If the matching edge lies at the end of the domino we get a tromino which matches the first domino shown on previous page. If the matching edge lies along one side of the domino, then by turning and moving it across the plane it can be matched with the second tromino.

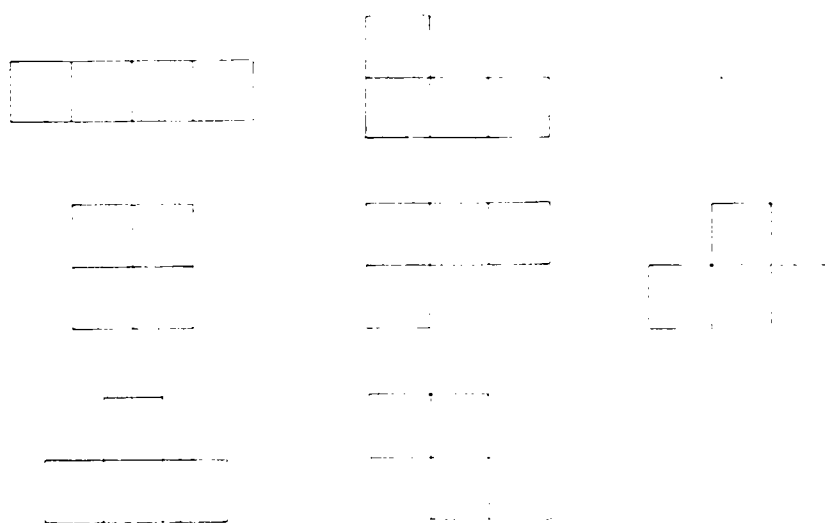


Thus there are exactly two distinct types of trominoes under a motion in the plane, the upright tromino and the right tromino.

Like the domino, the upright tromino matches itself if it is given a half turn or still even when its center. This is not true of the right tromino. It only matches itself after a full turn.

The next stage in the study is, clearly, the investigation of four-square configurations. They are called tetrominoes or four-squares. As in the previous case, the aim will be to find configurations which are distinct under the motions in the plane and to describe the motions which will take a given configuration into itself.

As before, all tetrominoes can be constructed by adjoining a square to a tromino. When all of the possibilities are investigated it turns out that there are seven distinct types.

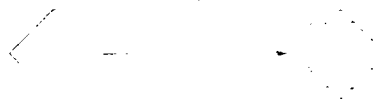


100

Math

1. A number line is shown below. The number 10 is marked on the line. The number 20 is marked on the line. The number 30 is marked on the line. The number 40 is marked on the line. The number 50 is marked on the line. The number 60 is marked on the line. The number 70 is marked on the line. The number 80 is marked on the line. The number 90 is marked on the line. The number 100 is marked on the line.

- The number 10 is marked on the line.
- The number 20 is marked on the line.
- The number 30 is marked on the line.
- The number 40 is marked on the line.
- The number 50 is marked on the line.
- The number 60 is marked on the line.
- The number 70 is marked on the line.
- The number 80 is marked on the line.
- The number 90 is marked on the line.
- The number 100 is marked on the line.

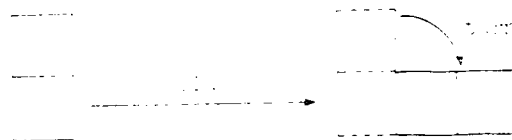


2. A number line is shown below. The number 10 is marked on the line. The number 20 is marked on the line. The number 30 is marked on the line. The number 40 is marked on the line. The number 50 is marked on the line. The number 60 is marked on the line. The number 70 is marked on the line. The number 80 is marked on the line. The number 90 is marked on the line. The number 100 is marked on the line.

3. A number line is shown below. The number 10 is marked on the line. The number 20 is marked on the line. The number 30 is marked on the line. The number 40 is marked on the line. The number 50 is marked on the line. The number 60 is marked on the line. The number 70 is marked on the line. The number 80 is marked on the line. The number 90 is marked on the line. The number 100 is marked on the line.

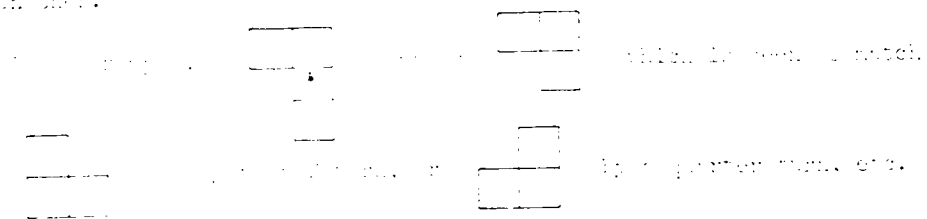
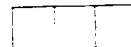
4. A number line is shown below. The number 10 is marked on the line. The number 20 is marked on the line. The number 30 is marked on the line. The number 40 is marked on the line. The number 50 is marked on the line. The number 60 is marked on the line. The number 70 is marked on the line. The number 80 is marked on the line. The number 90 is marked on the line. The number 100 is marked on the line.

5. A number line is shown below. The number 10 is marked on the line. The number 20 is marked on the line. The number 30 is marked on the line. The number 40 is marked on the line. The number 50 is marked on the line. The number 60 is marked on the line. The number 70 is marked on the line. The number 80 is marked on the line. The number 90 is marked on the line. The number 100 is marked on the line.



Insult: three-squares and one four-square. If the squares, by joining the
bottom edge, are to be made of something so that the edges match
perfectly, it is a four-square and a half, or it is another by
itself. It is a four-square and a half. It is very different
from a three-square and a half.

The skill is especially in trying different combinations, either individually or jointly, on either or on demonstrating solutions at the board or with the teacher or partner. They observe that they must begin by forming a two-type and that there are several possible places to add the third piece. (In fact, there are six.) Encourage them to try various possibilities and then to select the "winning" ones and eliminate those which can be made to fit another.

[illegible]

1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

[illegible][illegible][illegible]

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2. Do you think it is a good idea to have a national day of mourning?

This graphic will require additional material, and will be developed in
with the understanding by providing a graphic of the 11 square and
triangle divided into four squares and a triangle, three squares, 11
squares, triangle and one, etc. If it is understood that the 11 square is
a 11 square, then the 11 square is a 11 square, three squares, three-
square and one and three plus three-square, etc.

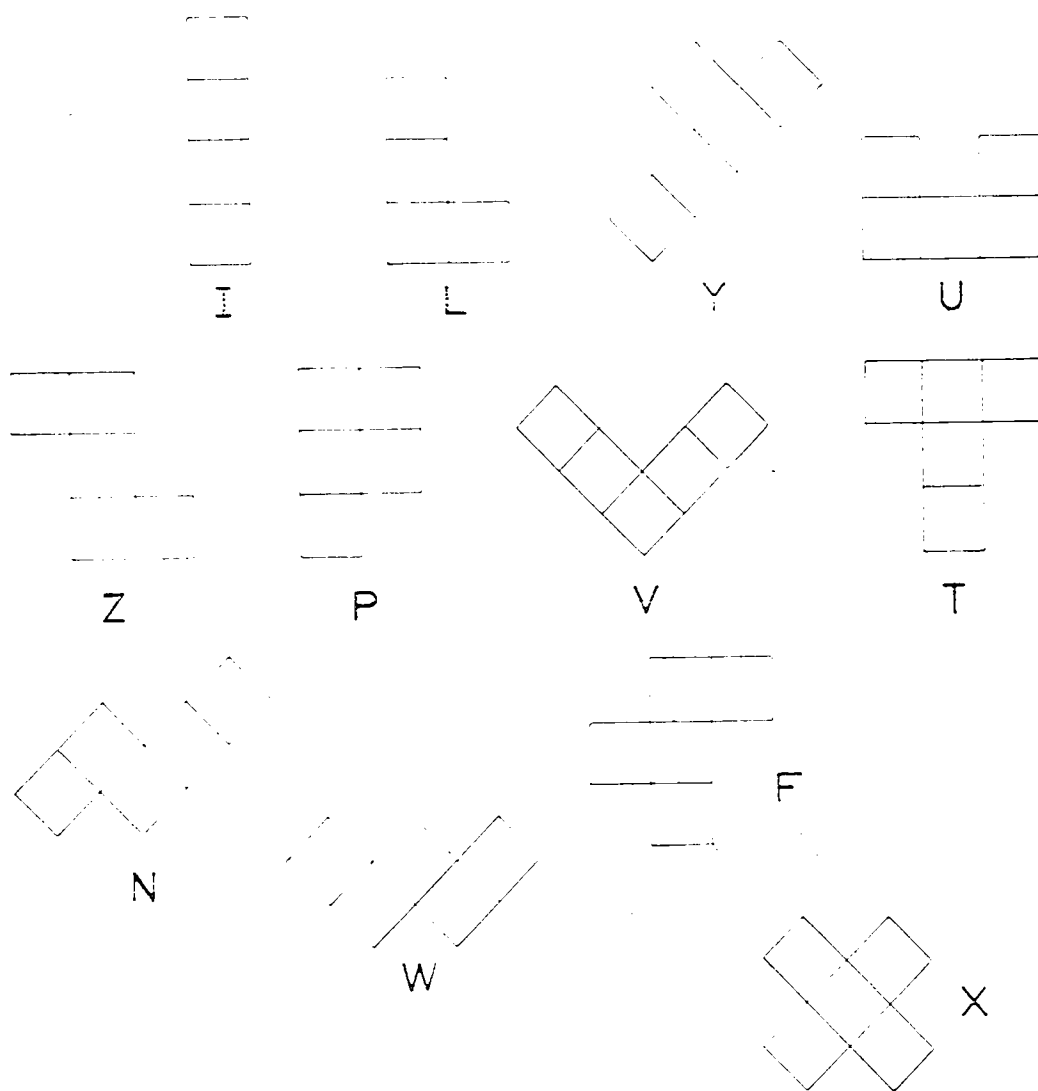
1. How do I determine if a person is a "white-collar" criminal?

1. Can you draw a rectangle with a diagonal three-square and straight three-square? Can it be done with two straight three-squares and a bent three-square? With two right three-squares and a straight three-square?

4. Turns: Play alternates between the two players. Two players take turns moving 1x = 1 square on the board. The player who places the last two-square wins. (All two-squares must be placed horizontally. It cannot be separated into two distinct squares).

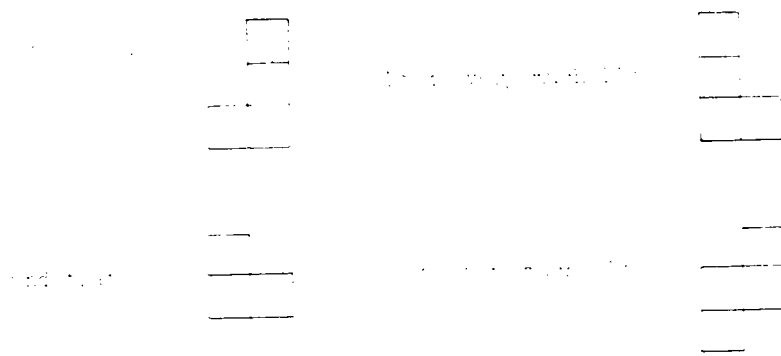
ACTIVITY III: Dominoes and I-forms to make letters, figures, and things

Materials: dominoes, I-forms, and a set of letter cards. (The letter cards should be made by cutting out the letters from a piece of paper, and then gluing them to a card.)



Dominoes and I-forms

Review the dominoes and I-forms. Show the children how to make the letters and figures. Ask if any pair looks similar. (If any two children will notice that



to confirm the results for that we have a set of three slides and one turn. We match the three slides and the turn and find that it is impossible to match the four squares.

Example: Can you move a four-square using only one slide to make it like another?

The children experiment with the slides and find that if they flip the figures over, the first slide the second and the third slide the fourth. We now have three different shapes which match the four squares. (1) slides, (2) turns and (3) flip.

Example: How many different shapes can a four-square have if we use slides, turns and flip.

After a few trials the children find that there are exactly five distinct four-squares. If they flip the first one they get a four-square from seven to five. The children experiment with the four-square by sliding, turning and flipping until they realize that there are exactly five distinct shapes.

It should be clearly understood that the number of different shapes depends on the motion allowed. If the rule says we can use slides and turns only, then seven different four-squares are formed: if the rule says we can use slides, turns and flip, then five different four-squares are formed.

Example: Using slides and turns, there are exactly two different three-squares (one and the other, and the right three-square). How many different three-squares are there using slides, turns and flip?

SPECIAL FIFTEEN AND GAMES

1. Can all the five-squares be made with a three-square and two-squares?

Again, the question is in that whether the five-square can be made with three-squares and two-squares.

It seems that all



can be.

Next, let us see if we can make a five-square with a three-square and a two-square. In other words, can we make a five-square with a three-square and a two-square? The answer is yes, and it is shown in the diagram.

On a four-square number of squares, it is possible to play with a three-square and a two-square. In other words, it consists of shading in three-squares and two-squares. It is also possible to play with a three-square and a two-square. In other words, it consists of shading in three-squares and two-squares. It is also possible to play with a three-square and a two-square. In other words, it consists of shading in three-squares and two-squares.

ACTIVITY III: SQUARES

Materials: A square (five-square) made of paper with
central and four corner squares removed.
(In this activity, the central square is the "hole".
The four corner squares are the "flaps".)

Learning Objectives

Example: How many times can the L five-square be turned so that it matches itself?
(A half turn)

Experimentation will show that the L five-square completely
around in a full turn (360 degrees) matches itself. This can be done by the child
in several ways. If a drawing of the configuration is on one piece of paper,
it may be traced on another piece of paper and the latter turned until the traced
figure exactly fits the original configuration. Alternatively, the configura-
tion may be cut out of the paper and placed on paper. Leaving the paper on
the table, the figure may be turned until it once again fits into its
original position.

When a half-turn and a full turn of the L five-square, one can
trace around the outline in an outline and then turn the outline until it
matches the original outline.

Example: How many times can the E five-square be turned so that it matches itself?
(A half turn)

In this case, experimentation will show that if a turn the figure half-
way around, it fits into its original position. The original figure.
March around a full turn (360 degrees) will match the original figure. The children
should discover that any figure can be made to match itself under a full
turn. The children will then be asked to take the E-square into a
matching figure.

For this activity, the children may be asked to half turn followed by another
half turn and see if it matches the original figure.

Example: How many times can the E five-square be turned so that it matches itself?
(A half turn)

[illegible]

Further evidence is given in the following pages. For example, in the child's story of the "finger" in which, as is shown, he has not seen the finger of Mani in the "finger", he has indicated that he has seen the end of a pointer that he has seen in the "finger" and he has seen it in the "finger" under the "finger", and he has seen it in the "finger".

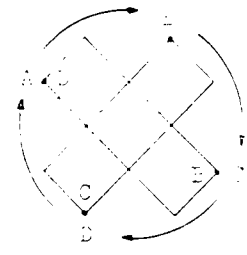
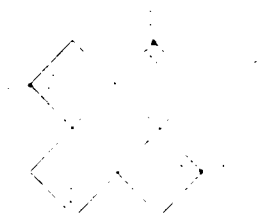
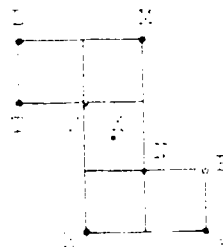


Figure 1

the two pointer four elements, A is matched with B, B is matched with C, C is matched with D, and D is matched with A.

I want to know how you feel about the situation.
 I want to know how you feel about the situation.



14. The following is a list of the half-ton, ten-ton, and twenty-ton, minimum loads. They can record their own weight, etc.

Figure 1. The effect of the number of nodes on the number of iterations required to reach the optimal solution for the 1000 nodes problem. The number of iterations required to reach the optimal solution increases as the number of nodes increases. The number of iterations required to reach the optimal solution for the 1000 nodes problem is approximately 1000.

[illegible]

1. The first step is to identify the variables. In this case, the variables are the number of people who attended the event and the number of people who did not attend the event. (You may wish to begin by identifying the variables in the data set.)

1000

[illegible]

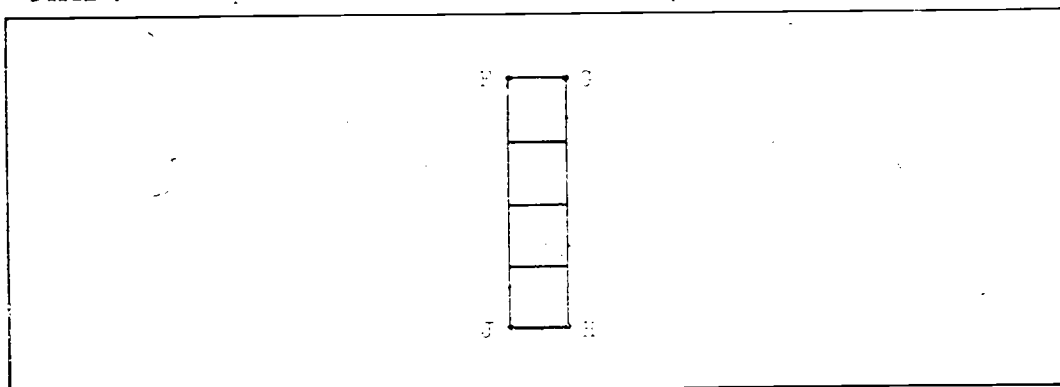
1. *Chlorophyll a* (Chl *a*)

1. 2. 3. 4.

1990

1. *Journal of the American Medical Association*, 2000; 283: 2686-2692.

Example: Which points match when the I five-square is flipped?



First, let the children, then verify, by flipping the figure. Here the children discover that there are two ways to flip the figure and the matching of points depends upon which way it is flipped. For example, if it is flipped across a vertical line through the center,

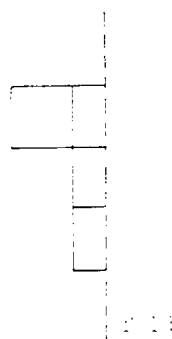
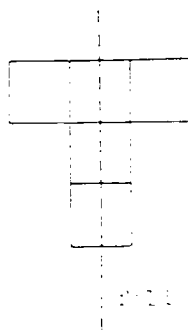
$G \rightarrow F$, $F \rightarrow G$, $J \rightarrow H$, $H \rightarrow J$.

But, if it is flipped across a horizontal line through the center,

$F \rightarrow J$, $J \rightarrow F$, $G \rightarrow H$, $H \rightarrow G$.

Example: Which of the five-squares can be folded so that the lines on one side fit exactly the lines on the other side?

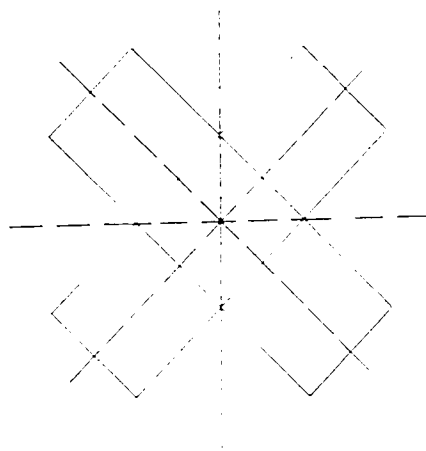
Probably figures drawn on tracing paper provide the best method of experimentation here. Experiment with a I five-square drawn on thin paper. If you fold down the center, the children see that the lines of one-half fit exactly the lines of the other half.



Experimentation shows that the I, the L and the X can be folded so that one-half fits the other, but not the I or the V. If the children compare the five-squares that are able to be flipped to match, they discover that those which have a folding line are exactly those which could be flipped to match. (None of them has a line of symmetry if and only if it can be flipped and moved into a matching position.) The children can also discover that an I five-square has two folding lines.

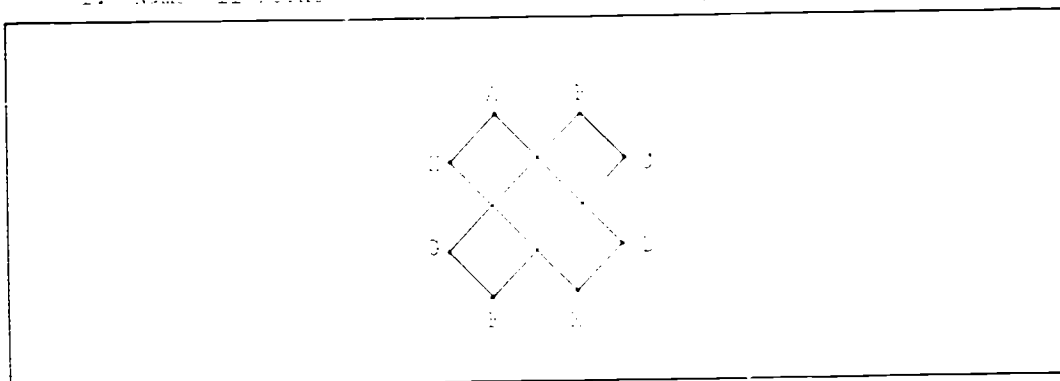
Purple: How many foldin lines does it have-4-?

Experimentation with paper shows that there are four lines of symmetry as shown below.



SPECIAL EXAMPLES: SYMMETRY OF PENTOMINOES

1. Name all points which are matched with A by flip.



Since there are four lines of symmetry, each flip (or fold) matches A with a different point.

$$A \rightarrow B, \text{ or } A \rightarrow C, \text{ or } A \rightarrow D, \text{ or } A \rightarrow E$$

2. Tell which points in the other three are matched by a flip with flip 1.

$$A \rightarrow B, C \rightarrow D, D \rightarrow E, E \rightarrow F, F \rightarrow G, G \rightarrow H, H \rightarrow A$$

3. If B is matched with C ($B \rightarrow C$), what point is matched with C ?

$$(C \rightarrow A \text{ by a different flip})$$

4. If C is matched with D ($C \rightarrow D$), what point is matched with D ?

$$(D \rightarrow A \text{ by a flip})$$

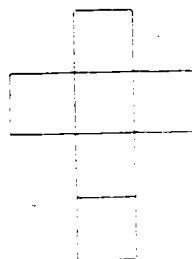
5. If D is matched with E ($D \rightarrow E$), what point is matched with E ?

$$(E \rightarrow A \text{ by a flip})$$

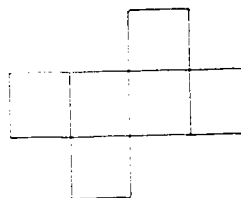
$$(A \rightarrow B, C \rightarrow D, D \rightarrow E, E \rightarrow F, F \rightarrow G, G \rightarrow H, H \rightarrow A)$$

ACTIVITY IV: Some Hexominoes

Puzzle: Six-squares are made by joining six of the squares. Two examples are:



and



Which can be made from a four-square and a two-square?

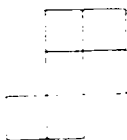
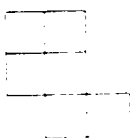
The first one, not the second one.

Puzzle: Which of the above six-squares can be made from two three-squares?

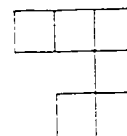
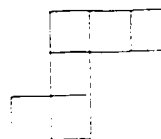
The second, not the first.

Puzzle: Which of these pairs of like-pieces are different shapes and which are the same shape? If they are the same, how can you move one to fit the other?

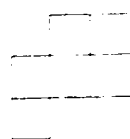
1.



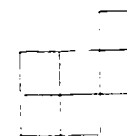
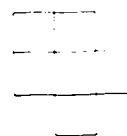
2.

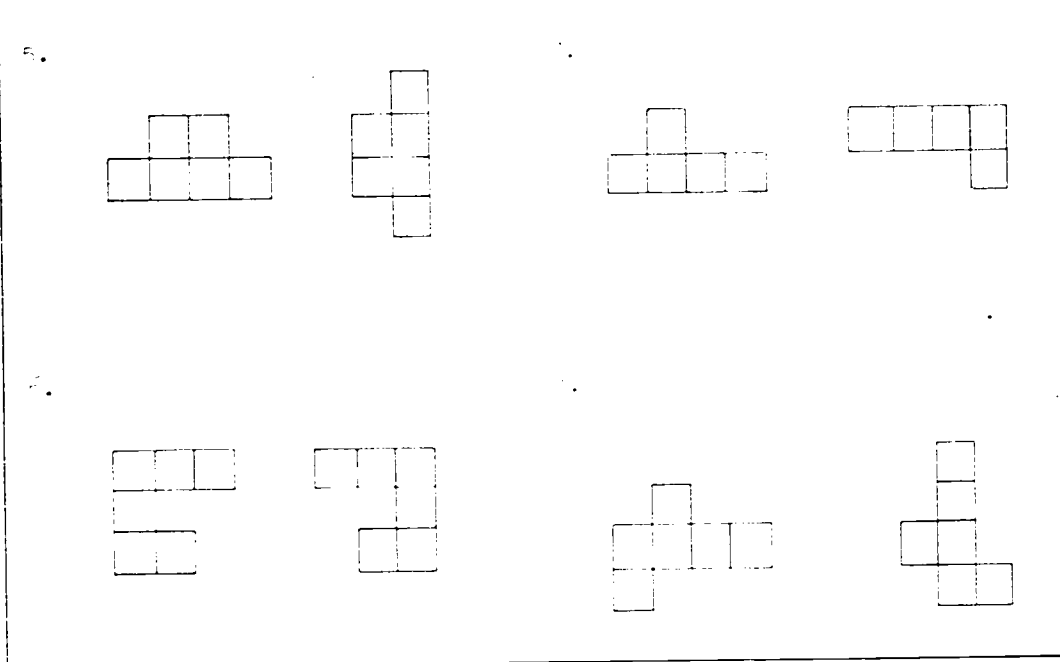


3.



4.



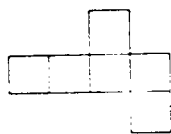


- (a) only, where 1111 is not a
- (b) only, where 1111 is not a
- (c) not a
- (d) not a
- (e) only, where 1111 is not a
- (f) only, where 1111 is not a
- (g) not a
- (h) only, where 1111 is not a

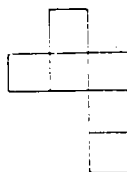
SPECIAL PUZZLES AND GAMES

1. Which of the six-squares can be made from a four-square and a two-square but not from two three-squares?

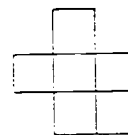
(a)



(b)



(c)



2. You have a 4×3 board. Can you completely cover the board using

- (a) a five-square, a six-square and a one-square?
- (b) using a three-square, a five-square and a four-square?
- (c) using a six-square, a four-square, and a two-square?
- (d) using a six-square, a five-square, and a two-square?
- (e) using a five-square, a four-square, and a one-square?

The children may select any of the configurations listed; for example, to answer the first question they try any of the five-squares with any six-square and a one-square. They try fitting them together in the manner of a big-box puzzle until they find a combination that exactly fits the 4×3 rectangle.

This may be played as a game by two children who take turns selecting. The first child to select the three pieces (polyominoes) that will fit the 4×3 rectangle wins.

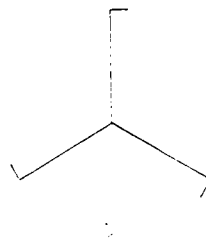
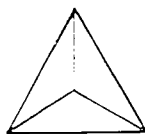
In experimenting with combinations, the child may become aware that there are some pieces that never fit. This is all too obvious that only a combination with a sum of 12 units could possibly fit. If he notices this, he may see immediately that the answer to (b) is "no" because the total number of squares is 13. Similarly he may try the combinations and finally conclude that nothing fits because there are too many squares. In either case, he may be able to apply these conclusions to question (c) and see that the answer must again be "no" because a five-square, a four-square, and a two-square give a total of only 12 units.

SIMMETRY

(Prepared by F. Linden)

BACKGROUND ON SIMMETRY

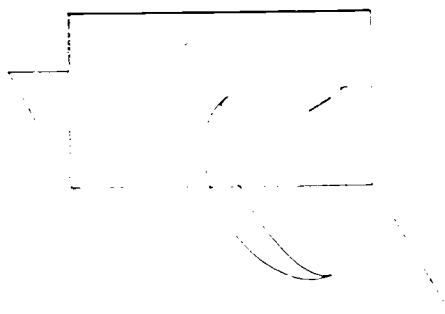
These patterns are symmetrical:



These patterns are not symmetrical:



Symmetrical patterns are those which look the same after being rotated, reflected or translated in some way. Patterns 3 and 4, for instance, look the same when rotated one-half turn. (To see this turn the page upside down.) Patterns 3 and 5 look the same when rotated one-third or two-thirds of a turn. Thus patterns 3, 4, 5, and 6 have rotational symmetry. Pattern 1, the lune, does not have rotational symmetry. (When rotated one-half turn, for instance, the lune looks different: the points are to the left instead of to the right.) The lune has reflective symmetry. If you place a mirror perpendicular to the paper with one edge along a horizontal line through the center of the lune as shown below:



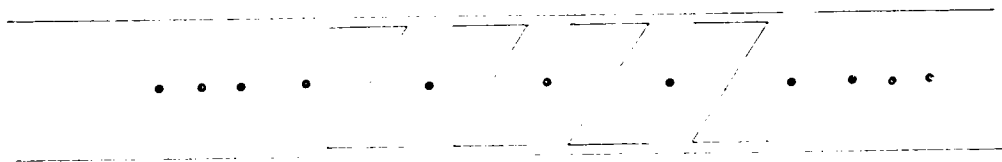
then the visible half of the line together with its reflection looks identical to the complete line. Since reflection is a reasonable word to describe the symmetry of the line, the horizontal line is the line's line of symmetry. The two halves of any figure folded along a line of symmetry match exactly. Pattern 1 has two lines of symmetry (vertical and horizontal lines through the center) and pattern 2 has the line of symmetry. A square has four lines of symmetry. Patterns 3 and 4 have no lines of symmetry (though, of course, they do have centers of rotational symmetry).

The definition of symmetry given above contains the key words: rotated, reflected and translated. We have discussed the first two. A figure is said to be translated if it is shifted in some direction without changing its orientation. Only infinite patterns can look the same after a translation. Imagine, for example, that this pattern is extended infinitely in both directions:



Since this pattern can be found if it is translated any whole number of units to the left or right, it has translational symmetry. (It also has reflection symmetry with respect to a horizontal center line.)

A second example is shown below:



In addition to its translational symmetry, this pattern also has rotational symmetry (it can be rotated 180 degrees). (Is it also, however, a helix?)

Other patterns, for example, the complex translated helix symmetric pattern,

are just a few of the many patterns that can be created. A systematic study of these patterns is called fractal geometry, a beautiful and rapidly developing branch of mathematics that has led to the rapid study of symmetry.

References

1. For ingenious art based on symmetry and: M. C. Escher,
"The Graphic Work of M. C. Escher," Meredith Press, New York,
1967.
2. For mathematical theory with many pictures and: L. Fejes Tóth,
"Regular Figures," Pergamon Press, New York, 1964.

GEOMETRY UNIT

(1) Definition by Example

Display a collection of symmetrical shapes (labeled "symmetrical") and a collection of unsymmetrical shapes (labeled "unsymmetrical"). (Suggested shapes are attached.) After the students have studied the two collections for a few minutes produce new shapes and ask them to be identified as symmetrical or unsymmetrical. Do not try to define symmetry in words. Include many kinds of symmetry (vertical, horizontal, rotational, etc.). E. g., .

(2) Working by Discovery (from definition)

Display a symmetrical shape. Hold it in front of it and pretend to manipulate it. Pretend to ask, "How do you do it?" (rotation.) (Show the shape is symmetrical.) It is a simple matter to tell. This reveals a definition: If a shape can be turned in any way without changing its appearance, it is symmetrical.

Now display a collection of shapes in the old way, including at least one unsymmetrical one and one with rotational symmetry. A symmetrical shape can be turned in any way without changing its appearance, but an unsymmetrical one cannot.

(3) Lines of Symmetry

Now display some symmetrical shapes and a collection of lines. Ask what the lines have in common. (They are all lines of the same kind. Lines of all kinds are called "lines.") Now display some lines of symmetry (fold lines) and ask what they have in common. (They are all lines of symmetry.)

7 SUGGESTED SHAPES

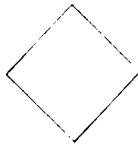
No symmetry:



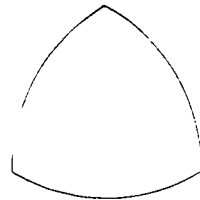
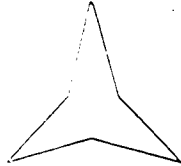
One line of symmetry:



Two lines of symmetry:



Three lines of symmetry:



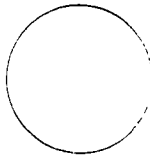
Four lines of symmetry:



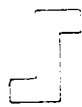
Five lines of symmetry:



∞ lines of symmetry:



2-fold rotation:



4-fold rotation:



(Mirror mounted on black.) Observe that if a mirror is placed along a line of symmetry, the visible half of the figure together with its reflection looks the same as the figure itself.

Observe that shapes with a line of symmetry fit into their outlines when flipped over. (Shapes with rotational symmetry fit into their outlines when turned like a pinwheel.) You have now tested for lines of symmetry three ways: (1) by folding, (2) by reflecting in a mirror and (3) by flipping. Establish in discussion that the three tests are equivalent. A good way to do this is to challenge the pupils to find a shape that passes one test (e.g., folding) but fails another (e.g., reflecting). Of course, this is impossible. Do some shapes have more than one line of symmetry?

(4) Work Sheet 1 (attached)

Lead and work sheet 1 for pupils to do in class. Go around class with mirror, holding it at right angles to the line of symmetry, to place mirror along mirror of line of symmetry.

(5) Homework

Find (English, black) shapes that are symmetrical with,

- (a) a line of symmetry.
- (b) two lines of symmetry.
- (c) three lines of symmetry.
- (d) rotational symmetry but not a line of symmetry.

(6) Game

Material: A 10x10 grid of squares, divided by a heavy line into two 5x5 halves. 10 numbered cards numbered each of such size as to cover exactly four squares of the grid.

Rules: Shapes are placed on the grid. Each square must have at most one card placed on it. Once a square is put down it is not to be used.

It is impossible to place the heavy center line, and no shape may overlap another shape which has been put down before, and no shape may extend outside the grid.

A player wins when the other player cannot find a place to put a shape.

Example 1: The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square.

Example 2: The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square.

Example 3: The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square.

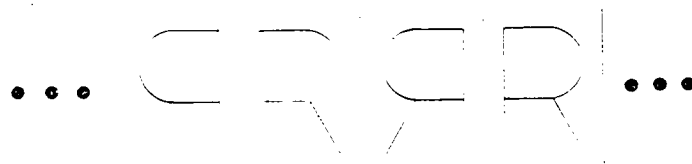
Example 4: The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square.

Example 5:

The figure shows a shape with rotational symmetry but no line of symmetry. The shape is a square. The figure shows a shape with rotational symmetry but no line of symmetry. The shape is a square. The figure shows a shape with rotational symmetry but no line of symmetry. The shape is a square.

Example 6:

The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square. The figure shows a shape with rotational symmetry. The shape is a square.



How does it compare with the previous work? Is it a theorem? Find out by looking up the definition of \mathcal{H}^1 in the paper by Federer and Fleming. (Theorem 2.10.1, p. 114 of [1]). What is the relationship of \mathcal{H}^1 to the \mathcal{H}^1 defined in [1]?

(1) Definition 1.1.1. The \mathcal{H}^1 norm.

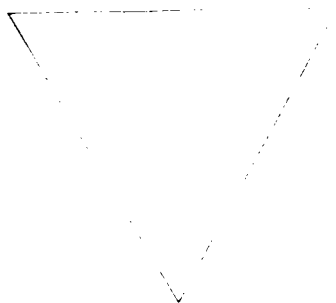
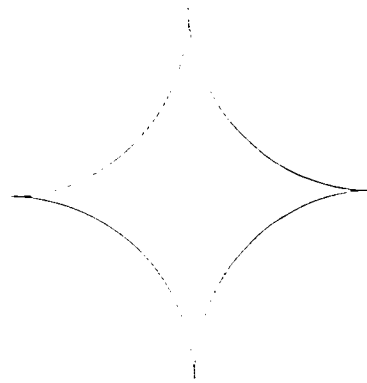
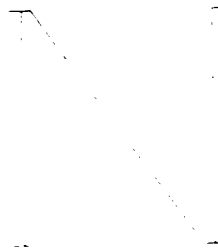
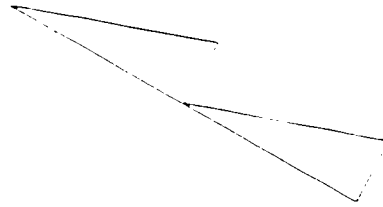
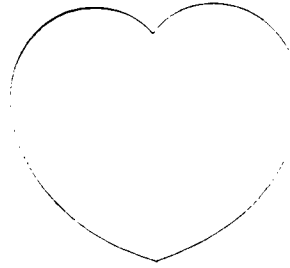
Let μ be a Borel measure on \mathbb{R}^n . Let f be a function on \mathbb{R}^n which is measurable with respect to μ . Let \mathcal{H}^1 be the space of functions f which are measurable with respect to μ . Let \mathcal{H}^1 be the space of functions f which are measurable with respect to μ . Let \mathcal{H}^1 be the space of functions f which are measurable with respect to μ .

(2) Definition 1.1.2. The \mathcal{H}^1 norm.

- (a) Let μ be a Borel measure on \mathbb{R}^n . Let f be a function on \mathbb{R}^n which is measurable with respect to μ .
- (b) Let μ be a Borel measure on \mathbb{R}^n . Let f be a function on \mathbb{R}^n which is measurable with respect to μ .
- (c) Let μ be a Borel measure on \mathbb{R}^n . Let f be a function on \mathbb{R}^n which is measurable with respect to μ .
- (d) Let μ be a Borel measure on \mathbb{R}^n . Let f be a function on \mathbb{R}^n which is measurable with respect to μ .
- (e) Let μ be a Borel measure on \mathbb{R}^n . Let f be a function on \mathbb{R}^n which is measurable with respect to μ .

WORK SHEET 1

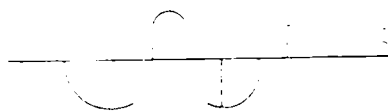
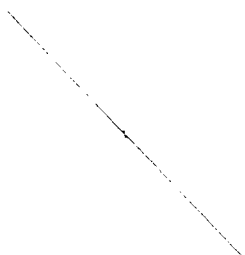
Draw all kinds of shapes you can.



WORKSHEET

Handwriting

Complete the letters on the lines and copy the letters on the lines.



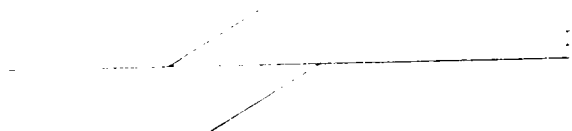
Report on trial in Seattle, Wash.

Item 1. The first trial, the first trial was the first trial. The trial was the first trial, the first trial was the first trial. Item 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

Item 2. The second trial, the second trial was the second trial. The trial was the second trial, the second trial was the second trial. Item 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

Item 3. The third trial, the third trial was the third trial. The trial was the third trial, the third trial was the third trial. Item 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

Item 4. The fourth trial, the fourth trial was the fourth trial. The trial was the fourth trial, the fourth trial was the fourth trial. Item 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.



Item 5. The fifth trial, the fifth trial was the fifth trial. The trial was the fifth trial, the fifth trial was the fifth trial. Item 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

Game: The game was played on a Vitruph, but a glassboard would be better. It required a lot of excitement and noise. I started out playing the (a) division strategy (a section involved in reflection) and switched to strategy (b) (a section involved in reflection) and switched to strategy (c) (a section involved in reflection). I played until they were out. Then play other strategies (i.e., other transformations). It would also even conclude that it is advantageous to respond. It is a fact that if you win if you win first and your opponent fails to play a geometry strategy, and it can be easily shown. Your winning demonstration will show that the student that is not yet found the result.

Even when the student knew how to win, it took considerable convincing to them to understand why the symmetry strategy was so important. It was only when they had understood it to the end. The second time they had to win. If the pattern of squares is symmetric then it is the pattern of squares. Therefore, if there is a square of squares, then there must also be a square (on the other side) of squares.

For the trial

It was found that the trial was not as well as the trial. The demand for the trial was not as high as the demand for the trial. The material will be presented and, if the teacher is not as well as the trial. In particular, the trial was not as well as the trial. The trial was not as well as the trial.

It was found that the trial was not as well as the trial. The trial was not as well as the trial.

It was found that the trial was not as well as the trial. The trial was not as well as the trial.

It was found that the trial was not as well as the trial. The trial was not as well as the trial.

pulling to the right. The ship should be driven forward. It is too
late now. The ship is already too far from the shore.

And the ship is already too far from the shore. The ship is
already too far from the shore. The ship is already too far from the shore.
The ship is already too far from the shore. The ship is already too far from the shore.

CLASSROOM ACTIVITIES BASED ON COMPUTER CONCEPTS

A COUNTING MACHINE

(Prepared by W. Charles and J. Hank Aaron)

INTRODUCTION

The purpose of this activity is to help students learn essential aspects of number representation in terms of counting on a computer-like classroom control. The initial purpose is to demonstrate how the computer can be applied to this activity:

1. The teacher will first present a picture of a counting machine with no numbers on it. In this picture (which should take 5-10 minutes), the emphasis is on getting the "machine" to perform correctly and in setting the class to identify the relation between the number entered and the appearance of the "machine parts."
2. When the machine is performing fairly accurately, the teacher asks the class to enter a number and to identify the number for any machine representation and the machine representation for any number (if it is possible). This phase should take about 5-10 minutes.
3. Finally, the teacher can pose questions which the students can answer by using the machine accurately. However, the questions can also be answered by using their own relationship between the representation and the machine representation of numbers. This exploring question is excellent for developing the kind of understanding which is often lacking in students.

NUMBER REPRESENTATION

First, let's look at the way the computer represents numbers. When we write 100, we mean that 1 is in the hundreds place, 0 is in the tens place, and 0 is in the units place. We can represent the number 100 as $1 \times 100 + 0 \times 10 + 0 \times 1$. We can emphasize this by writing $1 \times 10^2 + 0 \times 10^1 + 0 \times 10^0$ or $1 \times 10^2 + 0 \times 10^1 + 0 \times 10^0$. In all cases, this representation of numbers is correct. If we wanted to represent a number such as one thousand, we would write $1 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 0 \times 10^0$. The 1 thousand is represented as 1×10^3 or 1×1000 or $1 \times 10 \times 10 \times 10$.

...and the people of the world. If we could we represent
the people of the world. If we could we represent the people of the world:

[illegible]

ACTIVITY: 1A

Teacher says:

"Has anyone ever seen a counting machine?"

"Today we are going to make our own counting machine. In fact, the moving parts of our counting machine will be our fingers."

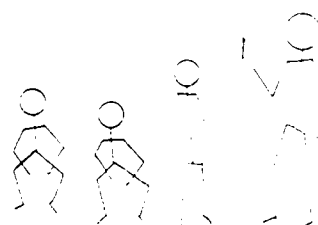
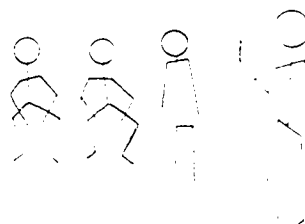
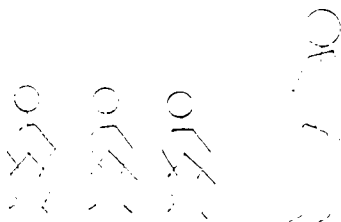
"Now let's see how the parts of our machine will move when it is counting."

"Now the machine will count to one."

"This is the way the machine shows the number one."

"Now the machine will count to two."

Classroom appearance:



Comments:

Observe students' experience counting out mechanical motion and parts of suggested counting machines.

Select three students and arrange in three chairs at front of room.

Write a tally mark on the board. Pass the chalk to the student seated in the first chair. Instruct the student to stand and pass the chalk to the student in the second chair. Instruct each student to pass the chalk until you get it from the student in the third chair.

It is important to establish the relationship between each number counted or recorded in the tally and the machine representation of each number.

Before counting to two, be sure that no one has changed position since you counted to one.

Write a tally mark on the board. Pass the chalk to the student standing at the first chair. Instruct him to change position and pass the chalk to the second student.

Teacher says:

Class:

Answer:

How many:

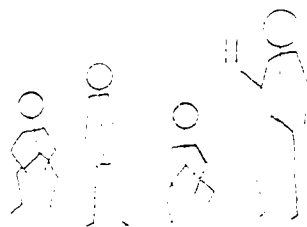
"This is the way the machine works. The number two."

"Now the machine will count to three."

"This is the way the machine works. The number three."

"This is the way the machine operates. I will write the number two on the board with a piece of chalk. This is the number which is being counted. I pass the chalk to the student at the first chair. He always carries his position and passes the chalk to the student at the next chair. This is done until all others have passed. I always watch the student who passed them the chalk. If the student who passed the chalk sat down, the next student would bring the chalk and pass it to the next student."

If the student who passed the chalk stood up, the machine would stop. The next student would bring the chalk and pass it to the next student. If the student who passed the chalk sat down, the next student would bring the chalk and pass it to the next student."



Now instruct the second student to change position and stand. The third student does again with the chalk, remains seated, and passes the chalk to the teacher.

Repeat the same procedure one more tally.

After instructing the machine for several tallies, then the rules for the motion of each part should be given.

As you can see the sitting down of a student is simply a signal for the next student to change position. This sitting down is like the carrying which takes place in adding.

It acts in the same way as the mileage indicator on a car: when the right hand dial moves from nine to zero, the next dial is advanced one digit.

Teacher says:

"Now let's start counting for each machine part and count to four."

"This is the way the machine counts the number three."

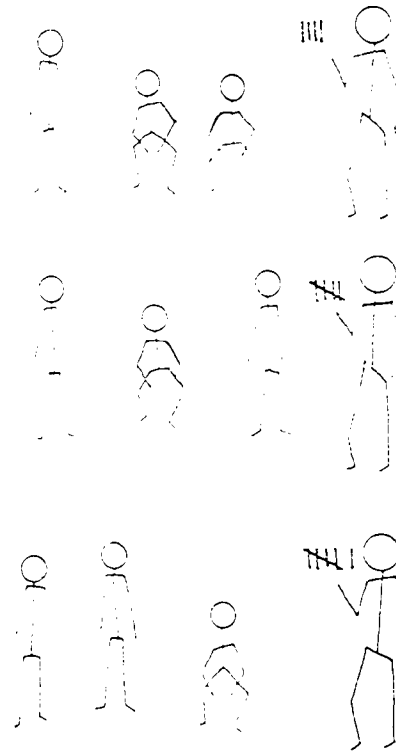
"Now let's start five."

"This is the way the machine counts the number five."

"And now six."

"This is the way the machine counts the number six."

Teacher's appearance:



Teacher says:

"Follow the machine part in counting several more tallies. Always count the number of tallies to see whether the particular machine part is working correctly. It is important for the children to know the machine's part is not to be for the machine itself. This is also an excellent way to involve the child in the process. You may want to count the machine part after you have succeeded in getting the machine to work properly. A second machine will ordinarily work properly much more quickly. In selecting the machine parts, it is necessary that the first student be able to keep the operation moving. Each machine part is important, but smooth counting machine operation is lost when the first machine part does not respond to the tallying."

ACTIVITY: Phase 1

Teacher says:

"Now let's see if we can tell what number the machine is naming. What number is the machine naming when it looks like this?"

"What about this?"

"What number is the machine naming when both these machine parts are standing?"

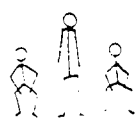
"What number is this?"

"And now this?"

"And this?"

"Now let's see if the machine will name a number we pick. Let's try two. What would it stand for?"

Classroom representation:



Comments:

You now control the parts of the machine to make the representations you want. If the students in the class are not able to identify the representations either by memory or by using the base two, you can always begin the counting again from zero until you get to the representation you want. Do not attempt to explain the relationships. Give the students an opportunity to discover whatever relationships they can for themselves by your careful selection of examples. Note that the suggested order in which the representations are presented emphasizes that two is the base and that the representations of each machine part are added.

The students in the class should be able to respond to these with little difficulty.

Teacher says:

"Let's try naming 18 on the machine. Who should be standing?"

"Who should be standing if we name seven on the machine?"

ACTIVITY: Phase 1

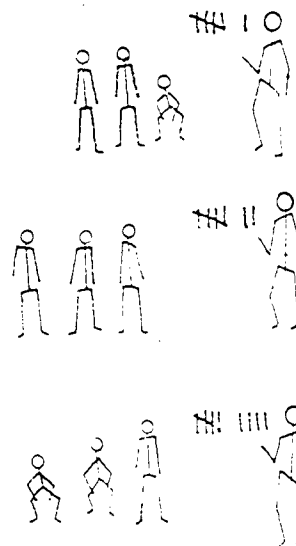
"How would the machine name the number nine?"

"This is not a satisfactory since this is also the way the machine named one. What could we do to the machine in order to be able to name nine in a different way than one?"

"Let's add another machine part."

"Now let's count from seven."

Classroom activity:



Comments:

You might let the class suggest numbers to represent on the machine. If you do this, and any number greater than seven is mentioned, you will be ready to move to the Phase 2 activity.

You are not ready to ask some of the questions which involve the students in extending their observations to new situations with the counting machine. Since the students are not likely to be able to answer these questions immediately, you should begin with a number representation they know -- such as seven -- and count to nine using the rules.

The students will ordinarily suggest that if another machine part is added to the machine the number nine can be named in a different way than one. Add another student to the machine part and begin counting from seven again.

Teacher's role:

Monitor the students' work and provide assistance when needed.

Classroom organization:

Students work in groups of four. Each group has a designated recorder, a designated checker, and two designated input/output operators.

Comments:

Here the students should be given a chance to guess. The correct answer is fifteen. You can handle the responses in a variety of ways. You can count and check the responses by simply following the rules for the machine, or you can explain the representations for each machine part.

You can now return to the Phase 1 type activity and ask for representations of numbers on the machine and for the numbers represented by the machine with the additional machine part.

Additional machine parts may be added and the many possibilities explored. It is impossible to predict all the directions which this activity will take. You should be open to the variety of responses from the students and they will be able to freely explore the relationships involved in the binary representation of the numbers on the machine.

• • • • •

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.



One interesting change in the rule set is to explore, if each machine part after the first in a change position only when the machine part is covered the chalk to him stands up, we have a reversal of the counting procedure. This new rule enables us to count backwards.

APPENDIX

Below are some questions to ask students about what you are unsure of to whether they have completed the objectives of the exercise described in the objectives.

- Obj. 1) 1. Arrange a machine reading so that it represents the number four. Ask each student individually, "What number is named by the machine?"
- Obj. 2) 1. Arrange the machine so that it represents the number eleven. Ask each student individually, "What number is named by the machine?"
- Obj. 3) 1. Give available machine parts with four machine parts. Ask each student individually, "Arrange the machine so that the machine named the number 1444."
- Obj. 4) 1. Give available machine parts with four machine parts. Ask each student individually, "Arrange the machine so that the machine named the number 144."
- Obj. 5) 1. Give available machine parts with four new and inexperienced machine parts. Write the numeral 4 on a sheet of paper and ask each student to arrange the machine so that the machine named the number 4. Ask each student individually, "What number is named by the machine?"

CLASSROOM ACTIVITIES BASED ON COMPUTER CONCEPTS

FINDING THE GREATEST COMMON DIVISOR

(Prepared by M. Jacobs and J. Hinkelmann)

After you have read the Fall and Winter papers, you will be able to organize a demonstration of a computer finding the greatest common divisor of two numbers using the children in your classroom as the participants.

GREATEST COMMON DIVISOR

But before we begin, let's find the greatest common divisor of two numbers -- sometimes known as the greatest common factor of two numbers. This will not be the procedure that will be followed in the computer demonstration, but it will provide good background for us. To know that 6 is a factor of 12 is the same as saying that 12 is a whole number times a whole number such that $6 \times 2 = 12$. 6 is a factor of 12 because there is no whole number which when multiplied by 6 gives 12.

Now we can write all of the factors of 12. List them.

Your list should include 1, 2, 3, 4, 6, and 12. These are all of the factors of 12. Now, write down all factors of 18.

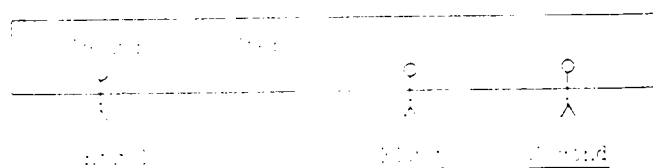
_____ . Your answer should be {1, 2, 3, 6, 9, 18}. What

factors of 12 and 18 are the same? _____ . 1, 2, 3, and 6 are the factors which are the same for 12 and 18. These are called the common factors. Similarly, the factors of these numbers is 6. 6 is known as the greatest common factor or the greatest common divisor. Let's find another greatest common divisor.

What is the greatest common divisor of 15 and 24? _____ . The answer is 3. Since the factors of 15 are {1, 3, 5, 15} and of 24 are {1, 2, 3, 4, 6, 8, 12, 24}, 3 is the only common factor of 15 and 24, and this is the greatest common factor or the greatest common divisor of 15 and 24.

FINDING GREATEST COMMON DIVISOR WITH COMPUTERS

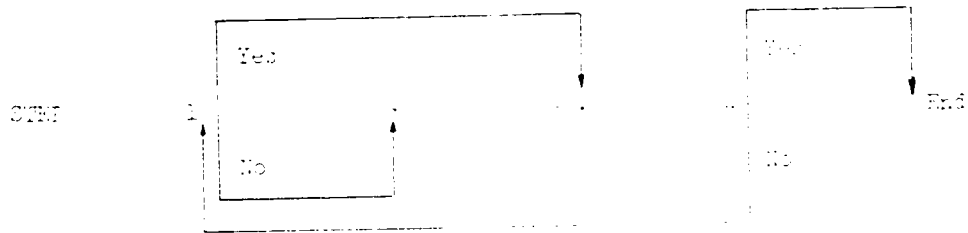
It is a good idea to have a computer in order to find the greatest common divisor of two numbers in an easy way. We could simply calculate these numbers -- called First, Second, and Third -- in the computer and get the result.



We will assume that the computer will find the greatest common divisor of the numbers First, Second, and Third and will write down the result. We will assume that the computer will find the greatest common divisor of the numbers First, Second, and Third and will write down the result of the calculation. We will assume that the computer will find the greatest common divisor of the numbers First, Second, and Third and will write down the result of the calculation. We will assume that the computer will find the greatest common divisor of the numbers First, Second, and Third and will write down the result of the calculation.

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DIAGRAM FOR FINDING THE GREATEST COMMON DIVISOR

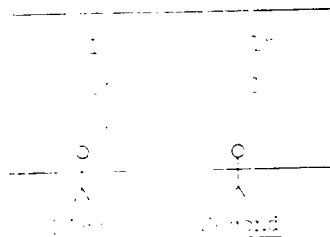


ALGORITHM FOR FINDING THE GREATEST COMMON DIVISOR

STEP	STATEMENT	ACTION
1	<u>First</u>	Let "First" be the number is greater than or equal to <u>Second</u> number, "Second" is smaller.
	<u>First</u> and <u>Second</u>	Let <u>Remainder</u> -- <u>First</u> written down <u>Remainder</u> number and <u>Second</u> written down <u>First</u> number.
2	<u>First</u>	Let <u>Remainder</u> and <u>Second</u> number from his and give the difference number.
3	<u>First</u>	Let "First" be the number is 0. "Second" is 1, not 0.

Control will call out the name of the student -- First or Second -- who is involved in action in that step. He will also read the action for that student for the step. Let's follow the activity with 1 and 18.

Control points to step 1 in the diagram and then says, "First," since the step is First and the student who is active in step 1. Since First's number is smaller than Second's, First says, "No." Control then follows the arrow which leads to step 2 in the diagram and finds her in pointing to step 2 in the diagram. Control says, "First and Second," since First and Second are both active in step 2. First then writes down 1, and Second writes down 18 in order to change the numbers of the steps in the diagram. Control then points to step 3 in the diagram and says, "First." First then Second's number from his and writes the first number. The chalkboard in front of First and Second would look like this at this point:



Control then points to step 4 in the diagram and says, "First." First calls, "No." and Second calls, "Yes." Control then follows the arrow which leads to step 5 in the diagram and says, "First." And First then says, "Yes," and Second says, "No." The two numbers.

The activity can be summarized in the following table:

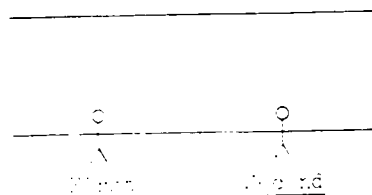
<u>Student</u>	<u>First</u>	<u>Second</u>	<u>Steps</u>
<u>First</u>	1	18	Begin
<u>Second</u>	18	1	1
<u>First</u>	1	18	2
<u>Second</u>	18	1	3
<u>First</u>	1	18	4
<u>Second</u>	18	1	5

Can you now begin to fill in the remaining responses in the table? What does First do at the point we stopped in the activity back at step 1? Does he call, "Yes" or "No"? _____. He must call, "No," since his number 6 is smaller than Second's number 12. Try completing the rest of the steps and compare with the completed table below.

<u>Control calls:</u>	<u>First</u> writes or calls:	<u>Second</u> writes or calls:	Step:
-	12	12	Begin
" <u>First</u> "	"No"		1
" <u>First</u> and <u>Second</u> "	6	1	2
" <u>First</u> "	6		3
" <u>First</u> "	"No"		4
" <u>First</u> "	"No"		5
" <u>First</u> and <u>Second</u> "	3	1	6
" <u>First</u> "	3		7
" <u>First</u> "	"No"		8
" <u>First</u> "	"No"		9
" <u>First</u> "	"No"		10
" <u>First</u> "	1		11
" <u>First</u> "	1		12
" <u>First</u> "	"No"		13
			End

The last number that Second has written down is the greatest common divisor. As we already know from our earlier examination, 3 is the greatest common divisor of 12 and 18.

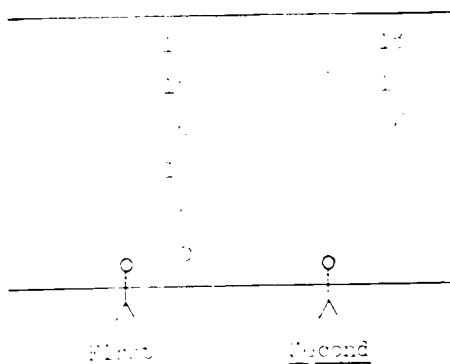
What would First and Second have written on the chalkboard in front of them in finding the greatest common divisor of 12 and 18? See if you can fill in the space on the record card below:



At this point in the activity First says, "No," since 6 is not smaller than _____. Then the activity on this page ends. Following the arrow skip step ____.

At this point in the activity, First says, "No," since the difference is _____. The activity on this page ends. Following the arrow skip step ____.

It should look like this:



Now try to do your own. Write out the table of responses for finding the greatest common divisor of 12 and 36.

Example table:

First writer
or caller:

Second writer
or caller:

Step:

Begin

12 18 24 30 36 42
1 2 3 4 5 6
12 18 24 30 36 42

1. The student will demonstrate the operation of the computer by playing the role of Control in the computer activity.

Initially, the teacher must make it clear to the expectation that students will play the role of Control in another computer program activity. Moreover, the student will write all the different and steps for procedures in mathematics.

APPENDIX

The teacher will give the student have required the immediate television for the activity, as well as in the activity by using the following questions. The teacher will give each student an exercise to be written, given these will be able to demonstrate with students.

Q1. 1. "In the network of the computer, can you find the greatest common divisor of 12 and 15?"

Q2. 2. "In the network of the computer, can you find the greatest common divisor of 12 and 15?"

Q3. 3. "Given 12 and 15, how does the student to be able to answer?" "How to demonstrate how you would be able to find the greatest common divisor of 12 and 15?"

LINEAR FUNCTION GAME

(Prepared by W. A. Dwyer and J. Larkin)

GENERAL DESCRIPTION OF VARIOUS LINEAR FUNCTION GAMES

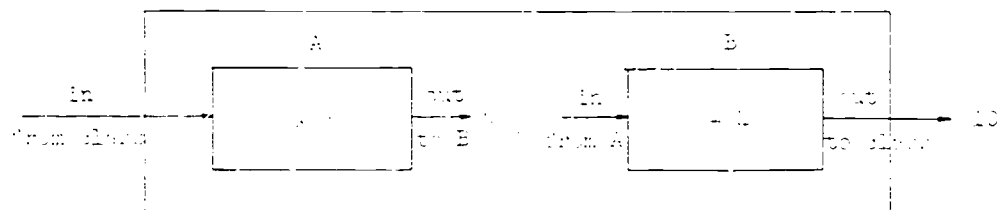
A variety of classroom activities can be devised in which all pupils can participate either as simple arithmetic "machines" or as operators of the operation of "machines" or such machines. The cluster of games described below uses the basic arithmetic transformations like "multiply by 4" or "add 1" as simple machines.

The fundamental elements of these games are the following:

1. A pupil, the machine, selects, or is assigned by the teacher, one of an agreed upon set of rules for transforming or converting a number into another number. The rule might be, for example, "multiply by 4."
2. A number of the kind selected becomes the input, which is communicated to the machine.
3. The machine, according to the rule, the output, into which his rule converts the input. Thus, in the example cited, if 5 is the input, the output is "20" as the output.
4. In "multiply by 4," i. e., the above, how can we identify the operation of the machine. In the example cited above this is very simple and in the machine is an arithmetic multiplier, one figure will always multiply. In all but the simplest arrangements the situation is more complicated.

Figure 1, Figure 2, and the simple machine described in 1. show a "simple" or a "linear" machine. In construction. One pupil, A, is an unknown number. The second pupil, B, is an unknown adder, i. e., he adds a fixed number to the first number. The third pupil, C, is a multiplier. The input number is multiplied by the multiplier, input is multiplied by 4. The latter takes the output of the machine and the first input is multiplied by the output.

For example:



Thus, in this case, the compound machine converts an input to a 10 output. Similarly, if 1 is fed into the machine, 7 comes out and if 5 is fed into the machine, 16 comes out.

The problem in 4, for the analyzer to identify each component, A and B, is doing. A number of third grade students are capable of discovering quickly that the machine can be identified by testing it with any input. They will usually immediately find a zero input gives away the value, B.

SUGGESTED GAME

SINGLE COMPONENT GAME - Grades 1 through 6

Function Game 1

In introducing the game, the teacher can play the role of the machine. Game 1 is a simple "add 2" game. The teacher announces that she is a machine which adds the same amount to whatever is that is given to her. The children are asked to give an input. The teacher then silently operates on the numbers and records the result in the form of an output. It would be advisable to fill the rest of input and output on the board as follows:

<u>Input</u>		<u>Output</u>
1	--	3
	--	5
	--	7

From the record of the game, it is clear that the teacher is adding 2 to each input. Except for the one who did, the need only one input to decide of the simple compound machine. And here. After the teacher had played the game several times, a student would be asked to play the role of the machine. Then the student would be allowed to take one of the roles and the teacher would play the role of the student who will record the input and output on the board. After the class has developed in concept, pair off

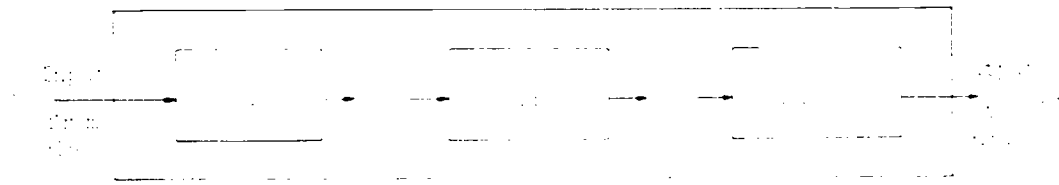
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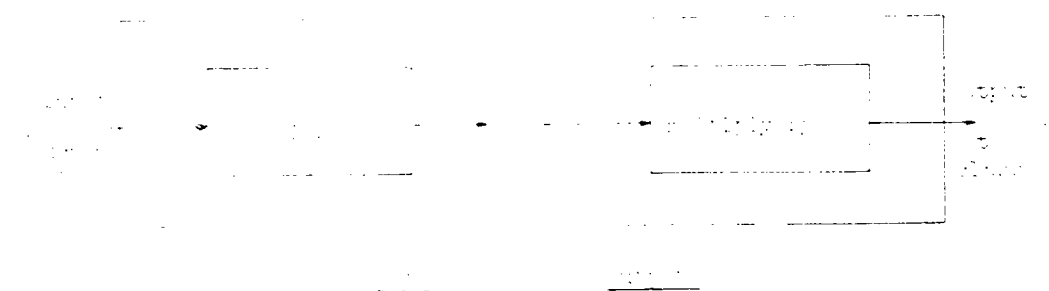
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The system is a closed-loop system. The output of the system is the output of the forward path, which is the output of the block $G(s)$. The output of the block $H(s)$ is fed back to the input of the block $G(s)$.

Example 1

The system is a closed-loop system. The output of the system is the output of the forward path, which is the output of the block $G(s)$. The output of the block $H(s)$ is fed back to the input of the block $G(s)$. The system is a closed-loop system. The output of the system is the output of the forward path, which is the output of the block $G(s)$. The output of the block $H(s)$ is fed back to the input of the block $G(s)$.



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[illegible]

Condition	10 years old (Open circles)	12 years old (Filled circles)
100% correct	~95%	~95%
75% correct	~85%	~85%
50% correct	~75%	~80%
25% correct	~45%	~75%

In the case of the United States, the Director of the Central Intelligence Agency is the principal official responsible for the conduct of the intelligence activities of the United States. The Director is also the principal official responsible for the conduct of the intelligence activities of the United States.

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Function Two

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Two

Three

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Function Three

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For example:



Students should be able to write down the sequence of operations and what address the operations are performed at. For example:

Input	Output	Input	multiply by 2
1	2	2	4
2	3	3	6
3	4	4	8
4	5	5	10

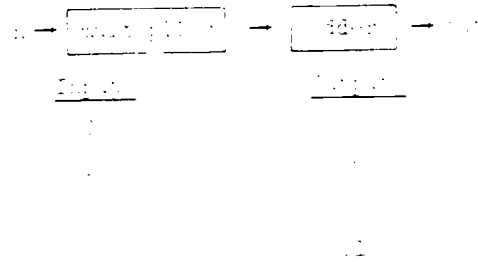
Students should be able to write down the sequence of the first machine. The second machine is not required. The second machine is immaterial.

Machine 2

Students should be able to write down the sequence of the first machine. The second machine is not required. The second machine is immaterial.

Students should be able to write down the sequence of the first machine. The second machine is not required. The second machine is immaterial.

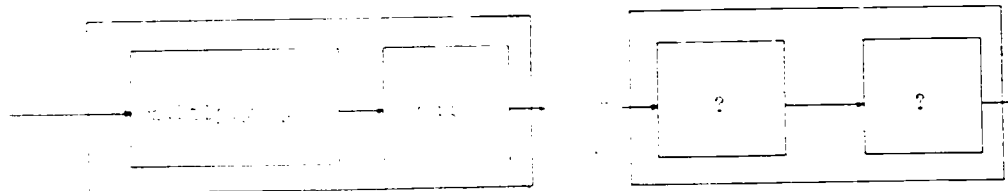
For example:



The children could then take a turn of "inputting" and then "adding".

The children could then build a "two component" machine which will take the two outputs from the first machine and insert them back into the original input. In this case, the children could be asked to place two numbers and then insert the two numbers into the original multiplier-adder machine. This case, the diagram indicated below on the next.

For example, the children could be asked to take the output of the first machine, which is 12, and insert it back into the original multiplier-adder machine. In diagram form:



Take the output of the first machine, and insert it in.

Input	Output
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Input	Output
1	0
2	1
3	2
4	3
5	4
6	5
7	6
8	7
9	8
10	9
11	10
12	11

Children could then take the type of "top-down" operation I mentioned. In this case, the children could be asked to take the output of the first machine, and insert it back into the original multiplier-adder machine. This is a more complex operation. The children could be more dynamic and creative.

In the first and last two multiplications, the third row is still
made as the sum of the first and second rows. In the third multiplication, the
third row is the sum of the first and second rows. In the fourth
multiplication, the third row is the sum of the first and second rows.

Figure 1 is a schematic representation of the experimental design. It shows a sequence of three steps: 1. A subject is presented with a stimulus (a face). 2. A response is recorded (a button press). 3. The subject is presented with a feedback stimulus (a face). The sequence is labeled 1, 2, and 3 respectively.

[illegible]

The figure consists of two separate line graphs, labeled (a) and (b), each plotting 'Rate of reaction' on the y-axis against 'Temperature' on the x-axis.

Graph (a) shows a straight line starting from the origin and increasing linearly with temperature. The line is labeled with 'A' at its upper end.

Graph (b) shows a curve that starts at the origin and increases with temperature. The curve is labeled with 'B' at its upper end. The slope of the curve increases as temperature increases, suggesting an activation energy barrier that is overcome at higher temperatures.

Journal of Management Education 36(8) 907-924
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[illegible]

THE UNIVERSITY OF CHICAGO

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1. The first part of the paper is devoted to the

study of the

problem of the existence of a solution of the

boundary value problem for the system of the partial differential equations of the second order in the domain G of the plane. The boundary conditions are given on the boundary of the domain G . The problem is solved in the case when the boundary conditions are given on the boundary of the domain G .

2. The second part of the paper is devoted to the

study of the problem of the existence of a solution of the boundary value problem for the system of the partial differential equations of the second order in the domain G of the plane. The boundary conditions are given on the boundary of the domain G .

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3. The third part of the paper is devoted to the

study of the

problem of the existence of a solution of the

boundary value problem for the system of the partial differential equations of the second order in the domain G of the plane. The boundary conditions are given on the boundary of the domain G .

The problem is solved in the case when the boundary conditions are given on the boundary of the domain G .

4. The fourth part of the paper is devoted to the

5

$$x_1, x_2, \dots, x_n \rightarrow 0$$

Figure 1

$$x_1, x_2, \dots, x_n \rightarrow 0$$

The first part of the proof is to show that the sequence x_1, x_2, \dots, x_n is bounded. This is done by showing that the sequence is bounded above and below. The upper bound is given by the first term of the sequence, x_1 , and the lower bound is given by the last term of the sequence, x_n .

$$x_1, x_2, \dots, x_n \rightarrow 0$$

$$x_1, x_2, \dots, x_n \rightarrow 0$$

The second part of the proof is to show that the sequence x_1, x_2, \dots, x_n is convergent. This is done by showing that the sequence is Cauchy. A sequence is Cauchy if for every $\epsilon > 0$, there exists a positive integer N such that for all $m, n > N$, $|x_m - x_n| < \epsilon$.

$$x_1, x_2, \dots, x_n \rightarrow 0$$

The third part of the proof is to show that the limit of the sequence is 0. This is done by showing that the sequence is bounded and convergent, and that the limit is 0.

$$x_1, x_2, \dots, x_n \rightarrow 0$$

The fourth part of the proof is to show that the sequence x_1, x_2, \dots, x_n is bounded. This is done by showing that the sequence is bounded above and below. The upper bound is given by the first term of the sequence, x_1 , and the lower bound is given by the last term of the sequence, x_n .

$$x_1, x_2, \dots, x_n \rightarrow 0$$

The fifth part of the proof is to show that the sequence x_1, x_2, \dots, x_n is convergent. This is done by showing that the sequence is Cauchy. A sequence is Cauchy if for every $\epsilon > 0$, there exists a positive integer N such that for all $m, n > N$, $|x_m - x_n| < \epsilon$.



Mathematics

Mathematics: Addition and Subtraction

1. Add 12 and 34.
2. Subtract 5 from 18.
3. Add 25 and 17.
4. Subtract 9 from 31.
5. Add 45 and 23.
6. Subtract 12 from 56.
7. Add 78 and 19.
8. Subtract 34 from 87.
9. Add 101 and 22.
10. Subtract 45 from 98.

Write the sum or difference of the numbers in the box.

Write the sum or difference of the numbers in the box.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Write the sum or difference of the numbers in the box.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Write the sum or difference of the numbers in the box.

Write the sum or difference of the numbers in the box.

Find the sum of the first 10 terms of the arithmetic sequence.

Answer: $S_{10} = 10 \cdot 1 + \frac{10(10-1)}{2} \cdot 2 = 10 + 10 \cdot 9 \cdot 2 = 10 + 180 = 190$

$$S_{10} = 10 \cdot 1 + \frac{10(10-1)}{2} \cdot 2 = 10 + 10 \cdot 9 \cdot 2 = 10 + 180 = 190$$

Find the sum of the first 10 terms of the arithmetic sequence.

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Example 3: Finding the Sum of an Arithmetic Sequence

Find the sum of the first 10 terms of the arithmetic sequence.

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$$S_{10} = 10 \cdot 1 + \frac{10(10-1)}{2} \cdot 2 = 10 + 10 \cdot 9 \cdot 2 = 10 + 180 = 190$$

Figure 1. The effect of the number of trials on the number of correct responses. The number of correct responses was significantly higher for the 10 trials condition than for the 5 trials condition. Error bars represent the standard error of the mean.

Figure 1. The effect of the concentration of the *Agrobacterium* suspension on the transformation efficiency of *Agrobacterium* strains. The *Agrobacterium* strains were incubated with the plant explants for 24 h. The explants were then cultured on the selective medium. The number of explants transformed was counted. The results are the mean \pm SD of three independent experiments. * indicates a significant difference ($p < 0.05$) between the control and the treated explants.

[illegible]

2

When he finished work with 2×3 multiplication tables, the children might not have noted that the ratio of the number in column 1 to the number in column 2 is constant. Also the ratio of the number in row 1 to the number in row 2 is constant. By the fifth or sixth grade the teacher can encourage a child to do a similar, following, 3×4 table and find the same.

1. $\frac{1}{2}$ 2. $\frac{1}{2}$ 3. $\frac{1}{2}$ 4. $\frac{1}{2}$ 5. $\frac{1}{2}$ 6. $\frac{1}{2}$ 7. $\frac{1}{2}$ 8. $\frac{1}{2}$ 9. $\frac{1}{2}$ 10. $\frac{1}{2}$

3. A natural number n is called *squarefree* if it is not divisible by any square of a prime. For example: 1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 26, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 49, 50, 51, 53, 54, 55, 57, 58, 59, 61, 62, 65, 66, 67, 69, 70, 71, 73, 74, 77, 78, 79, 82, 83, 85, 86, 87, 89, 91, 93, 94, 95, 97, 98, 99, 101, 102, 103, 105, 106, 109, 110, 113, 114, 115, 116, 117, 119, 121, 122, 123, 125, 126, 127, 129, 130, 131, 133, 134, 135, 137, 138, 139, 141, 142, 143, 145, 146, 147, 149, 150, 153, 154, 155, 157, 158, 159, 161, 162, 163, 165, 166, 167, 169, 170, 173, 174, 175, 177, 178, 179, 181, 182, 183, 185, 186, 187, 189, 190, 193, 194, 195, 197, 198, 199, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 241, 242, 243, 245, 246, 247, 249, 250, 251, 252, 253, 255, 256, 257, 259, 260, 261, 262, 263, 265, 266, 267, 269, 270, 271, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 285, 286, 287, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 301, 302, 303, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 337, 339, 340, 341, 342, 343, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 359, 360, 361, 362, 363, 365, 366, 367, 369, 370, 371, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 425, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 445, 446, 447, 449, 450, 451, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 463, 465, 466, 467, 469, 470, 471, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 501, 502, 503, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 674, 675, 676, 677, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 701, 702, 703, 704, 705, 706, 707, 709, 710, 711, 712, 713, 714, 715, 716, 717, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 746, 747, 749, 750, 751, 752, 753, 754, 755, 756, 757, 759, 760, 761, 762, 763, 764, 765, 766, 767, 769, 770, 771, 772, 773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 790, 791, 792, 793, 794, 795, 796, 797, 799, 800, 801, 802, 803, 804, 805, 806, 807, 809, 810, 811, 812, 813, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 826, 827, 829, 830, 831, 832, 833, 834, 835, 836, 837, 839, 840, 841, 842, 843, 844, 845, 846, 847, 849, 850, 851, 852, 853, 854, 855, 856, 857, 859, 860, 861, 862, 863, 864, 865, 866, 867, 869, 870, 871, 872, 873, 874, 875, 876, 877, 879, 880, 881, 882, 883, 884, 885, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 897, 899, 900, 901, 902, 903, 904, 905, 906, 907, 909, 910, 911, 912, 913, 914, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 931, 932, 933, 934, 935, 936, 937, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 952, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 973, 974, 975, 976, 977, 979, 980, 981, 982, 983, 984, 985, 986, 987, 989, 990, 991, 992, 993,

Figure 1 consists of three sub-graphs labeled (a), (b), and (c), each showing the percentage of correct responses (Y-axis, 0 to 100) across three conditions (X-axis: 1, 2, 3). The groups are: (a) 100% correct, (b) 75% correct, and (c) 50% correct. The graphs show that performance is generally higher for the 100% correct group and decreases as the condition number increases.

Group	Condition 1	Condition 2	Condition 3
(a) 100% correct	~95%	~90%	~85%
(b) 75% correct	~85%	~75%	~65%
(c) 50% correct	~75%	~65%	~55%

[illegible]

• *Chlorophyll a* and *Chlorophyll b* were determined by the method of Lichtenthaler and Whistler (1973). The *Chlorophyll a* and *Chlorophyll b* contents were expressed as mg g⁻¹ of dry weight.

Figure 1. The effect of the concentration of the *Agrobacterium* strain on the transformation efficiency of *Agrobacterium* strain 1024. The *Agrobacterium* strain 1024 was cultured in the YEA medium at 28°C for 24 h. The cell concentration was adjusted to 10⁸ cells/ml. The cell suspension was mixed with the cell suspension of the *Agrobacterium* strain 1024 at the ratio of 1:1, 1:2, 1:3, 1:4, 1:5, 1:10, 1:20, 1:30, 1:40, 1:50, 1:60, 1:70, 1:80, 1:90, 1:100, 1:200, 1:300, 1:400, 1:500, 1:600, 1:700, 1:800, 1:900, 1:1000, 1:2000, 1:3000, 1:4000, 1:5000, 1:6000, 1:7000, 1:8000, 1:9000, 1:10000, 1:20000, 1:30000, 1:40000, 1:50000, 1:60000, 1:70000, 1:80000, 1:90000, 1:100000, 1:200000, 1:300000, 1:400000, 1:500000, 1:600000, 1:700000, 1:800000, 1:900000, 1:1000000, 1:2000000, 1:3000000, 1:4000000, 1:5000000, 1:6000000, 1:7000000, 1:8000000, 1:9000000, 1:10000000, 1:20000000, 1:30000000, 1:40000000, 1:50000000, 1:60000000, 1:70000000, 1:80000000, 1:90000000, 1:100000000, 1:200000000, 1:300000000, 1:400000000, 1:500000000, 1:600000000, 1:700000000, 1:800000000, 1:900000000, 1:1000000000, 1:2000000000, 1:3000000000, 1:4000000000, 1:5000000000, 1:6000000000, 1:7000000000, 1:8000000000, 1:9000000000, 1:10000000000, 1:20000000000, 1:30000000000, 1:40000000000, 1:50000000000, 1:60000000000, 1:70000000000, 1:80000000000, 1:90000000000, 1:100000000000, 1:200000000000, 1:300000000000, 1:400000000000, 1:500000000000, 1:600000000000, 1:700000000000, 1:800000000000, 1:900000000000, 1:1000000000000, 1:2000000000000, 1:3000000000000, 1:4000000000000, 1:5000000000000, 1:6000000000000, 1:7000000000000, 1:8000000000000, 1:9000000000000, 1:10000000000000, 1:20000000000000, 1:30000000000000, 1:40000000000000, 1:50000000000000, 1:60000000000000, 1:70000000000000, 1:80000000000000, 1:90000000000000, 1:100000000000000, 1:200000000000000, 1:300000000000000, 1:400000000000000, 1:500000000000000, 1:600000000000000, 1:700000000000000, 1:800000000000000, 1:900000000000000, 1:1000000000000000, 1:2000000000000000, 1:3000000000000000, 1:4000000000000000, 1:5000000000000000, 1:6000000000000000, 1:7000000000000000, 1:8000000000000000, 1:9000000000000000, 1:10000000000000000, 1:20000000000000000, 1:30000000000000000, 1:40000000000000000, 1:50000000000000000, 1:60000000000000000, 1:70000000000000000, 1:80000000000000000, 1:90000000000000000, 1:100000000000000000, 1:200000000000000000, 1:300000000000000000, 1:400000000000000000, 1:500000000000000000, 1:600000000000000000, 1:700000000000000000, 1:800000000000000000, 1:900000000000000000, 1:1000000000000000000, 1:2000000000000000000, 1:3000000000000000000, 1:4000000000000000000, 1:5000000000000000000, 1:6000000000000000000, 1:7000000000000000000, 1:8000000000000000000, 1:9000000000000000000, 1:10000000000000000000, 1:20000000000000000000, 1:30000000000000000000, 1:40000000000000000000, 1:50000000000000000000, 1:60000000000000000000, 1:70000000000000000000, 1:80000000000000000000, 1:90000000000000000000, 1:100000000000000000000, 1:200000000000000000000, 1:300000000000000000000, 1:400000000000000000000, 1:500000000000000000000, 1:600000000000000000000, 1:700000000000000000000, 1:800000000000000000000, 1:900000000000000000000, 1:1000000000000000000000, 1:2000000000000000000000, 1:3000000000000000000000, 1:4000000000000000000000, 1:5000000000000000000000, 1:6000000000000000000000, 1:7000000000000000000000, 1:8000000000000000000000, 1:9000000000000000000000, 1:10000000000000000000000, 1:20000000000000000000000, 1:30000000000000000000000, 1:40000000000000000000000, 1:50000000000000000000000, 1:60000000000000000000000, 1:70000000000000000000000, 1:80000000000000000000000, 1:90000000000000000000000, 1:100000000000000000000000, 1:200000000000000000000000, 1:300000000000000000000000, 1:400000000000000000000000, 1:500000000000000000000000, 1:600000000000000000000000, 1:700000000000000000000000, 1:800000000000000000000000, 1:900000000000000000000000, 1:1000000000000000000000000, 1:2000000000000000000000000, 1:3000000000000000000000000, 1:4000000000000000000000000, 1:5000000000000000000000000, 1:6000000000000000000000000, 1:7000000000000000000000000, 1:8000000000000000000000000, 1:9000000000000000000000000, 1:10000000000000000000000000, 1:20000000000000000000000000, 1:30000000000000000000000000, 1:40000000000000000000000000, 1:50000000000000000000000000, 1:6000000

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11

INTRODUCTION. It is a common occurrence for teachers to have students who are fascinated with mathematical puzzles but who show little interest in the usual textbook exercises. Furthermore, even those students who faithfully do their problem assignments are rarely stimulated and challenged by the problems themselves. These observations suggest that the effectiveness of problem assignments would be very greatly increased if the usual drill-type exercises could be systematically replaced by puzzle-type problems and games which make use of the same mathematical skills. However, the construction of such problems and games is not easy, and most of the development has been directed toward recreational uses. Nevertheless, many educators feel that such material has a great potential for educational use, and have urged that a concentrated effort be made to develop mathematical games and puzzle-problems for this purpose. A proposal along these lines was made to the School Mathematics Study Group which has broad interests in mathematics curriculum development. After obtaining approval from the National Science Foundation, NSF, agreed to support a two-week conference to explore the possibilities for the development of mathematical games and puzzle-problems which would be appropriate for educational use. The basic aim of the conference would be to generate ideas. There would be no construction of mathematical concepts or preparation of text material.

THE CONFERENCE. The conference was held at Stanford University during the period June 7 - June 14, 1963 with the following mathematicians participating:

Professor R. P. Dilworth, California Institute of Technology
Dr. Walter Jacobs, Department of Defense
Professor William Lister, The State University of New York
Dr. Frank Sinden, Bell Telephone Laboratories
Professor C. J. Titus, University of Michigan
Mr. Herbert Wills, University of Illinois
Mr. Robert Wirtz, Carmel, California

All of the participants have extensive interests in the area of mathematical recreation and most of them have been involved in a variety of curriculum development activities.

The formal sessions of the Conference were essentially brainstorming sessions in which a variety of ideas for puzzle-problems and games were proposed by the participants for consideration by the group as a whole. The items which survived after detailed discussion and criticism were assigned to conference participants to be written up for the final report. On the average one such session was held each day. The remainder of the time was devoted to an extensive examination of the literature on mathematical recreations; to modifying, extending, and translating known recreational items in order to make them suitable for educational use, and to preparing reports on the results of these endeavors. The principal output of the Conference is the collection of working papers which is attached as an appendix to this report.

CONCLUSIONS AND RECOMMENDATIONS. First of all, it is clear that the development of an entirely new class of puzzle-problems or games is a very difficult undertaking. The abilities required are not unlike those required in developing an entirely new line of mathematical research. The essential

Feature which distinguishes good puzzle-problems from ordinary textbook problems is the sense of challenge, novelty, and in some cases, surprise which they contain. Likewise, an effective game must present a natural, attractive challenge to the players. But it is exactly these features which characterize first-class mathematical research. Hence, it is to be expected that outstanding contributions to the art of puzzle-problem construction will be rare. However, it also seems to be the case that the discovery of new ideas will frequently lead to a wide variety of puzzles and games. The many puzzles and games which have developed from S. DeLoomb's invention of polyhedra illustrate this point. During the relatively short period of the conference, some novel ideas for puzzle-problems and games were produced. However, it will take further development and experimentation to determine how effective and how broadly applicable the ideas are. In any case, it was the unanimous feeling of the participants that a joint effort such as this is most effective way to stimulate ideas in this area. They further recommended that similar conferences of this type be held from time to time to encourage creative activity in this area and to make use of the ideas and talents of other people. It was felt that a two-week period was about the right length of time since after a fortnight of intensive effort the wells of creativity appear to begin slowly run dry.

Many of the working papers do not present entirely new ideas for puzzle-problems and games but rather give ideas for the development of known material in order to increase its potential for educational use. This is clearly a very important activity in such a conference and in many cases much imagination and resourcefulness is required.

No attempt was made to present the material in polished form. The working papers serve merely to get the ideas down in a preliminary form suitable for later experimentation and modification for use in curricular materials.

All members of the Conference are agreed that the next stage in the development of this material should be experimental testing on an item by item basis. This will require a good teacher as well as a mathematician. Furthermore, simply turning the material over to a good teacher will not be enough since the objectives which the inventor had in mind may very well be lost in the process of adapting the item to classroom use. Perhaps the best arrangement will be to have the experimentation done by a team of two -- a good teacher and a mathematician. If the mathematician involved is not the man who prepared the item, so much the better. Under any circumstances it is the consensus of the Conference that until there has been such experimentation, the material should not be widely circulated. In particular, it should not be made available to textbook writers in its present form.

After experimental testing, the next stage should be the development of some sample text material incorporating the tested items. This could be a responsibility of a writing team. On the other hand, it is the feeling of the Conference participants that this should be approached in an imaginative way. For example, rather than trying to fit puzzle-piece items into the traditional text format, it might be much more effectively to design the text material around the problems and experiments themselves, so that the learning of mathematical facts and the development of mathematical skills is a natural by-product of the interest in these activities.

Finally, the participants wish to express their appreciation for the cheerful and hearty cooperation of the SMSC staff.

K. P. Dilworth, Chairman

APPENDIX

The working papers produced by the conference participants are collected in this appendix. It must be emphasized that these papers were prepared in order that there would be a permanent record of the ideas generated during the conference. No attempt was made to polish the presentation nor was any effort made to be complete and detailed in the discussion of the item. Rather the motivation was to get the basic ideas down in writing as quickly as possible and to get on to the generation of more ideas. Furthermore, the form of the presentation was that which was most convenient for the particular participant involved, and no effort was made to get uniformity.

It should be noted that the working papers vary from short accounts of very particular items to broad descriptions of whole classes of games and puzzles. The grade level extends over the primary and secondary spectrum. In many cases, the activity could be profitable for primary students while an insightful analysis could only be carried out by advanced secondary students. For example, Nim can be played by primary students, but they are hardly able to analyse the game. Finally, it should be mentioned that the items fall into three categories -- classroom activities, individual puzzle-problems, and games. Some of the items naturally fall into more than one category, but a rough distribution of the items is as follows:

Classroom presentation: 1, 2, 3, 6, 11, 10, 21, 25

Puzzle-problems: 10, 14, 15, 17, 19, 21, 22, 23, 24

Games: 1, 4, 5, 7, 8, 9, 12, 16, 18, 23, 26, 27

A rough classification of the items according to grade level is as follows:

Grades 1-4: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 17, 18, 20, 26

Grades 5-8: 1, 2, 3, 4, 6, 10, 11, 13, 15, 16, 17, 18, 19, 21, 22,

23, 24, 25, 26, 27

Grades 9-12: 5, 15, 17, 19, 21, 22, 24, 27.

THE CHINESE ECONOMY

The Chinese economy has been growing rapidly since the late 1970s. This growth has been driven by a combination of factors, including reforms in the agricultural sector, the establishment of special economic zones, and the implementation of market-oriented reforms. The government has played a significant role in guiding the economy, but it has also allowed for a degree of market competition. This has led to a rapid increase in the country's GDP and a significant improvement in the standard of living for its citizens.

One of the key factors in the Chinese economy's growth has been the reforms in the agricultural sector. In the late 1970s, the government implemented the household responsibility system, which allowed farmers to sell their surplus produce on the open market. This led to a significant increase in agricultural output and a decrease in food shortages.

Another important factor in the Chinese economy's growth has been the establishment of special economic zones. These zones were established in the early 1980s and were designed to attract foreign investment and technology. The government offered a range of incentives to foreign investors, including tax breaks and the ability to repatriate profits. This led to a rapid increase in foreign investment in the Chinese economy.

Finally, the implementation of market-oriented reforms has also played a significant role in the Chinese economy's growth. The government has gradually introduced market competition in a range of sectors, including manufacturing and services. This has led to a significant increase in efficiency and productivity in the Chinese economy.

The Chinese economy's growth has been a remarkable achievement. It has transformed the country from a poor, agrarian society into a major economic power. This growth has also led to a significant improvement in the standard of living for its citizens. However, there are still challenges facing the Chinese economy, including the need to further reform the financial system and to address the issue of income inequality.

ARITHMETIC FUNCTION MACHINES

A brief survey of several machines are suggested by the small sample listed below. They should, incidentally, provide practice in basic arithmetic skills, but the focus is primarily on (1) properties of numbers with respect to operations, (2) the function idea, and (3) the problem of inferring a model from experimental data.

I. Linear function machines (grades 4-6)

A student (the "Machine") picks a linear function, e.g., multiply by 2 then add 1. The class feeds inputs to the machine which then announces the output. The problem is to identify the machine's operation.

II. Composite function machines (grades 4-6)

The teacher assigns to one student a multiplier (say 2) and to another an addend (say 1). The class does not know which is a multiplier nor what numbers have been assigned. The class feeds up the students in order until they stop inputs. A, B, who plays his output to B who operates the machine and gives the final output to the class.



The problem is to infer what each machine is doing. The idea of putting machines end to end, or several machines, defined as those in which the multiplier is A, will come up naturally here.

III. Remainder machines (grades 4-6)

In a remainder machine, one divides x (say 1). For every positive integer x , the machine computes the remainder after dividing by n and announces it to the class. The problem is to identify the divisor.

The problem is to save the sums (when wn is less than say 100) for the case where wn remainder divides until it is identified.

24. A. 123.456 789 1011 1213 1415 1617 1819 2021 2223 2425 2627 2829 3031 3233 3435 3637 3839 4041 4243 4445 4647 4849 5051 5253 5455 5657 5859 6061 6263 6465 6667 6869 7071 7273 7475 7677 7879 8081 8283 8485 8687 8889 9091 9293 9495 9697 9899 100101 102103 104105 106107 108109 110111 112113 114115 116117 118119 120121 122123 124125 126127 128129 130131 132133 134135 136137 138139 140141 142143 144145 146147 148149 150151 152153 154155 156157 158159 160161 162163 164165 166167 168169 170171 172173 174175 176177 178179 180181 182183 184185 186187 188189 190191 192193 194195 196197 198199 200201 202203 204205 206207 208209 210211 212213 214215 216217 218219 220221 222223 224225 226227 228229 230231 232233 234235 236237 238239 240241 242243 244245 246247 248249 250251 252253 254255 256257 258259 260261 262263 264265 266267 268269 270271 272273 274275 276277 278279 280281 282283 284285 286287 288289 290291 292293 294295 296297 298299 300301 302303 304305 306307 308309 310311 312313 314315 316317 318319 320321 322323 324325 326327 328329 330331 332333 334335 336337 338339 340341 342343 344345 346347 348349 350351 352353 354355 356357 358359 360361 362363 364365 366367 368369 370371 372373 374375 376377 378379 380381 382383 384385 386387 388389 390391 392393 394395 396397 398399 400401 402403 404405 406407 408409 410411 412413 414415 416417 418419 420421 422423 424425 426427 428429 430431 432433 434435 436437 438439 440441 442443 444445 446447 448449 450451 452453 454455 456457 458459 460461 462463 464465 466467 468469 470471 472473 474475 476477 478479 480481 482483 484485 486487 488489 490491 492493 494495 496497 498499 500501 502503 504505 506507 508509 510511 512513 514515 516517 518519 520521 522523 524525 526527 528529 530531 532533 534535 536537 538539 540541 542543 544545 546547 548549 550551 552553 554555 556557 558559 560561 562563 564565 566567 568569 570571 572573 574575 576577 578579 580581 582583 584585 586587 588589 590591 592593 594595 596597 598599 600601 602603 604605 606607 608609 610611 612613 614615 616617 618619 620621 622623 624625 626627 628629 630631 632633 634635 636637 638639 640641 642643 644645 646647 648649 650651 652653 654655 656657 658659 660661 662663 664665 666667 668669 670671 672673 674675 676677 678679 680681 682683 684685 686687 688689 690691 692693 694695 696697 698699 700701 702703 704705 706707 708709 710711 712713 714715 716717 718719 720721 722723 724725 726727 728729 730731 732733 734735 736737 738739 740741 742743 744745 746747 748749 750751 752753 754755 756757 758759 760761 762763 764765 766767 768769 770771 772773 774775 776777 778779 780781 782783 784785 786787 788789 790791 792793 794795 796797 798799 800801 802803 804805 806807 808809 810811 812813 814815 816817 818819 820821 822823 824825 826827 828829 830831 832833 834835 836837 838839 840841 842843 844845 846847 848849 850851 852853 854855 856857 858859 860861 862863 864865 866867 868869 870871 872873 874875 876877 878879 880881 882883 884885 886887 888889 890891 892893 894895 896897 898899 900901 902903 904905 906907 908909 910911 912913 914915 916917 918919 920921 922923 924925 926927 928929 930931 932933 934935 936937 938939 940941 942943 944945 946947 948949 950951 952953 954955 956957 958959 960961 962963 964965 966967 968969 970971 972973 974975 976977 978979 980981 982983 984985 986987 988989 990991 992993 994995 996997 998999 10001001 10021003 10041005 10061007 10081009 10101011 10121013 10141015 10161017 10181019 10201021 10221023 10241025 10261027 10281029 10301031 10321033 10341035 10361037 10381039 10401041 10421043 10441045 10461047 10481049 10501051 10521053 10541055 10561057 10581059 10601061 10621063 10641065 10661067 10681069 10701071 10721073 10741075 10761077 10781079 10801081 10821083 10841085 10861087 10881089 10901091 10921093 10941095 10961097 10981099 11001101 11021103 11041105 11061107 11081109 11101111 11121113 11141115 11161117 11181119 11201121 11221123 11241125 11261127 11281129 11301131 11321133 11341135 11361137 11381139 11401141 11421143 11441145 11461147 11481149 11501151 11521153 11541155

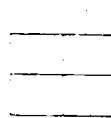
1. "Linear" functions take the form $f(x) = ax + b$ where a and b are constants. The graph of a linear function is a straight line. For example, the graph of $f(x) = 2x + 3$ is a straight line with a slope of 2 and a y-intercept of 3. The graph of a linear function is a straight line.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

EXERCISES AND DISCUSSIONS

1. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

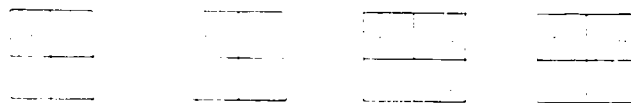
2. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?



3. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

4. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

5. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?



6. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

7. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

8. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

9. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

10. A student has a number of marbles. He has 10 more than 3 times as many as he has in his right hand. How many marbles does he have in his left hand?

It is not possible to win with 100% of the squares (row, column, diagonal) empty, and the player who moves first is at a disadvantage. The player who moves first is at a disadvantage because the first player has to leave the first square empty, and the second player can always win by taking the first square.

The first player can win by making a move that leaves the first square empty, and the second player can win by taking the first square.

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

100% of the squares empty

The game is played on a 3x3 grid. The player who moves first is at a disadvantage because the first player has to leave the first square empty, and the second player can win by taking the first square.

| | | | | | | | | |
|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |

The game was originally played on a 3x3 grid, but at first the player who moves first is at a disadvantage because the first player has to leave the first square empty, and the second player can win by taking the first square.

With 100% of the squares empty, the sum of 1 or 2 entries can win, and with 3 entries only the row, column, and diagonal sums win, hence the game is equivalent to tic-tac-toe. The question arises whether all magic squares yield games equivalent to tic-tac-toe, or whether in particular the sum of 1 entries or the sum of 2 entries will not form a row, column, or diagonal sum win.

Obviously, the game is played on a 3x3 grid, but at first the player who moves first is at a disadvantage because the first player has to leave the first square empty, and the second player can win by taking the first square.

h. Wires

1. The first part of the paper is devoted to a discussion of the

main results.

2. The second part of the paper is devoted to a discussion of the

main results of the paper.

3. The third part of the paper is devoted to a discussion of the

main results of the paper.

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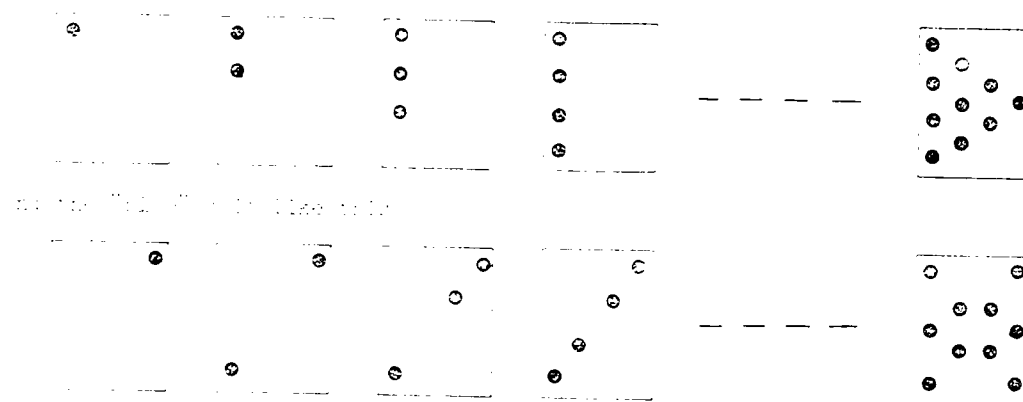
[illegible][illegible]

1. *Phragmites australis* (Cav.) Trin. ex Steud.

NUMBER CONCEPT WITH NUMERALS

In addition to the use of concrete objects or pictures to teach number concepts, children should be learning to write the numerals which are adapted from their knowledge of the objects. In the very early stages, a sample of these is given below. These numerals have the advantage of poor suitability for general classroom activity but may be useful as small group activities which are confined to the classroom. They provide a general context in which basic numeral skills are developed, and their somewhat repetitive effort in introduction is justified.

In order that these numerals be suitable in the first year, a printed special format is suggested. A first example is considered below. It is of course possible to use the numerals in the various columns. For example the "10" will be used in the first column.



These numerals are suitable for use in the first year, and can be further developed by using only with numerals in the various columns.

NUMBER CONCEPTS

The material of the numerals in the format described above may be used in this section. The first objective in the earliest stage is to develop children's knowledge of the numerals and their use with respect to number. Numerals are not at first written, but are used. Later, as the child's knowledge of numerals develops, the numerals should be written and the number-numeral associations.

• *Journal of the American Medical Association*, 2000; 283: 2639-2644

1. The following information is being furnished to you for your information only. It is not intended to constitute an offer of insurance or any other financial product. Insurance, securities, and other financial products are sold by separate entities. The information is being provided to you for your information only.

Almost all the rules of conventional bridge are covered, though in play, they are not very realistic. Obviously this will not be described here. A few single or interesting rules are left outside the simplified book. The 12 card for each player are divided into 10: and the upper 10 cards of 1 suit is explained by 11. In the simplest version the "trick" can be eliminated and a bonus of 100 points sent to each player. The game is best played on a board with parallel rows of 100 points which each player moves a point for each trick won or if like a domino game, points.

UNIT: NUMBER LINE; PAGE: 10; DATES: 1/2/78

A. Introduction

1. The subject of number lines is probably as important for the children as writing the way they express complex problems by using pictures of number lines.

2. This subject is a good basis for a whole series of exercises which can be done individually, in pairs, and at the end of the unit, and then the class.

3. I have made some activities which are suitable for the first step of the number line program; this is already a good start to the children's understanding, and it is already clear that it will be taught in a very interesting way.

B. An example

1. The first activity is "Number Line", which asks the students to draw a line, parallel to the horizontal line, and then to draw some of these lines.

2. The second activity is "Number Line", and shows the state (from arm to arm, from arm to arm) of the number line, and an effective signal. The third activity is "Number Line", and shows the state of the number line, and an effective signal. The fourth activity is "Number Line", and shows the state of the number line, and an effective signal.

3. The fifth activity is "Number Line", and shows the state of the number line, and an effective signal. The sixth activity is "Number Line", and shows the state of the number line, and an effective signal. The seventh activity is "Number Line", and shows the state of the number line, and an effective signal.

4. The eighth activity is "Number Line", and shows the state of the number line, and an effective signal. The ninth activity is "Number Line", and shows the state of the number line, and an effective signal. The tenth activity is "Number Line", and shows the state of the number line, and an effective signal.

[illegible][illegible][illegible]

Interpretation: The 10 smaller areas in the 100' x 100' grid on the river side, during 1978. The 100' sections along in a 10' x 10' grid side, in the order: One, Two, Three, Four, Five and Six. The same process is repeated, from north to south of the 10' grid side.

1. The situation relative to him.

2. He then indicates "forward" his arm, and then makes a quarter turn to the right, as he is facing the road.

3. Then, after looking at road he has just come up the one in front of him and then the right one. If the one in front is rising his arm, he makes a half turn to the right. If the one in front has dropped his arm, he makes a half "forward" his arm, and then makes a quarter turn to the right.

4. Mr. Tikhonov and Tikhonov's wife drew all three positions and Tikhonov began. At this point, they withdrew, by the positions of their arms, a considerable number of men from them.

... The three men who were the leaders, and all three had been in the
... and had been in the position of their work. They are still alive, and
... and are still alive.

Example 1: If the initial number is 1, then the number of children at the end of the game is the number of children.

| | <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> |
|------------------------|----------|----------|----------|----------|----------|
| Initial number --- low | low | low | low | low | low |
| After 1st game --- low | low | low | low | low | low |
| After 2nd game --- low | low | low | low | low | low |

Extending the number sequence "1" is represented, then at the end of any game the number represented is the sum of the initial number and the number of children, provided that this sum has not exceeded 5. When the class understands the game, the students may be asked to predict what the results will be in a situation starting with 2.

Extending: If the initial number from an initial number, the rule is that "Position" (up or down) "reversed" arms when the position in front raises his arm, and simply "up" when the position in front drops his arm. It will take a bright student to understand the effect of this change in the rule.

If a teacher is not with arms, the same type of game can demonstrate or train; the game can be played with children represented by

- 1 - both arms down
- 2 - both arms up
- 3 - both arms up

Example: A Base-10 Addition Calculator

Introduction: In previous sessions, students have learned the principles of the operation of the calculator.

Participants: Two teams of four students, one team of four students, and one team of four students, respectively, of the calculator. Three students, three students, twenty-sevens students, thirty-sevens students.

Description: Each team performs as a calculator machine, and attempts to add a series of six numbers as rapidly as possible. The teams may operate in pairs, or more complex ways. They may be given freedom to choose their own method of operation, ranging from deliberate and reliable to rapid and impractical methods. The students are aware of the likelihood of a mistake. A class discussion of possible methods will help to bring out the complexity of addition at the sub-operation level.

Procedure: The details of operation follow principles similar to those described in connection with the Binary Counter Game. Each position exhibits a digit in base 10 by holding up a corresponding number of arms, and adds another digit, counting forward from the current position. For example, if one arm is up corresponding to the digit "1" and the digit "1" is to be added, the position raises the one arm and silently counts "one", then drops both arms and silently counts "two". Having reached the digit to be added, he is done. The next student position must have been watching, and must add "1" as a carry at the point where the other has dropped both arms.

At the end of the process for adding a base 10 number to one already represented by arm positions of the team is as follows: the step-by-step method of addition:

1. The first position adds the units digit of the new number, which is written on the blackboard. He signals that he is done by making a quarter turn to face the blackboard.

2. The second position makes a quarter turn to face the blackboard, and adds "1" if there was a carry, as signaled by the first dropping both arms. He continues to add the tens digit of the number on the blackboard, and when he is done he signals that he is done.

3. This continues until thirty-sevens has finished his addition. At this point he calls out "Three". Then turns to face the board; all others turn to face him; and the team is set to start adding the next number.

_____ : _____, the pressure is not felt. The feeling is
not a feeling of pressure, but a feeling of being pushed.

_____ : _____, the pressure is not felt. The feeling is
not a feeling of pressure, but a feeling of being pushed.

_____ : _____, the pressure is not felt. The feeling is
not a feeling of pressure, but a feeling of being pushed.

_____ : _____, the pressure is not felt. The feeling is
not a feeling of pressure, but a feeling of being pushed.

_____ : _____, the pressure is not felt. The feeling is
not a feeling of pressure, but a feeling of being pushed.

As a sequence of steps is as follows: these are the different possibilities, and the actual sequence, which proceeds as in the illustration below:

| <u>STEP</u> | <u>CHARACTERISTIC</u> | <u>ACTION</u> |
|-------------|--------------------------|--|
| | <u>Teacher</u> | Holds 1, and reads "1" to <u>Number</u> . |
| | <u>Number</u> | Points to <u>Units</u> the digit on line named by <u>Teacher</u> . |
| | <u>Units</u> | Points to number held the digit named by <u>Number</u> . |
| | <u>Sum</u>
<u>Yes</u> | Calls "Y" if sum is less than 10, "N" if sum is greater than or equal to 10. (retains units digit of sum.) |
| | <u>Sum</u>
<u>No</u> | Points 1 to number held. |
| | <u>Sum</u>
<u>Yes</u> | Calls "Y" if sum is less than 10, "N" if sum equals 10. (retains units digit of sum.) |
| | <u>Overflow</u> | Displays 1. |
| | <u>Counter</u> | Adds 1 to number held. |
| | <u>Counter</u> | Calls "Y" if sum is less than 10, "N" if sum equals 10. (the total number of digits to be added.) |

When Yes is received, the final total is given as the pair of digits held by Sum and Units, provided that Overflow still shows 0. If he displays 1, the error number differs from that held by some multiple of 100.

The participant will hold a card showing the step numbers at which he takes action, and the nature of the corresponding action. Even in this case, it will not be clear the teacher will add name as the number of the current step is indicated.

The sequence of calls and numbers held for the first three steps of the activity. It is easy to verify that this sequence corresponds to the sequence of calls that Number list holds: 1, 1, 1, ...

| <u>LINE</u> | <u>DEFINITION</u> | <u>INITIAL</u> | <u>TEST</u> | <u>VALUE</u> | <u>OPERATION</u> |
|-------------|-------------------|----------------|-------------|--------------|------------------|
| 1 | | | 1 | 0 | 0 |
| 2 | Define "1" | | 1 | 0 | 0 |
| 3 | 1 | Define "2" | 1 | 0 | 0 |
| 4 | 1 | | 1 | 0 | 0 |
| 5 | 1 | | 1 | 0 | 0 |
| 6 | 1 | | 1 | "No" | 0 |
| 7 | Define "3" | | 0 | 1 | 0 |
| 8 | "11" | | 0 | 1 | 0 |
| 9 | | Define "4" | | | 0 |
| 10 | | | 1 | 11 | 0 |
| 11 | | | 1 | "Yes" | 0 |
| 12 | | | 1 | 1 | 0 |
| 13 | | | "No" | 1 | 0 |
| 14 | Define "5" | | 1 | 1 | 0 |
| 15 | "111" | | 1 | 1 | 0 |
| 16 | | Define "6" | | 1 | 0 |
| 17 | | | 1 | 11 | 0 |
| 18 | | | 1 | "Yes" | 0 |
| 19 | | | 1 | 1 | 0 |
| 20 | | | "No" | 1 | 0 |
| 21 | Define "7" | | 1 | 1 | 0 |
| 22 | "1111" | | 1 | 1 | 0 |

This line will be non-zero if N is greater than 5, and at the next cycle will be zero. The total "1" of the first 5 digits corresponds to the pair of digits being Tens and Units.

Extension: The class may undertake to extend the procedure to handle the addition of numbers of several digits. A more difficult project is to provide the capability to subtract as well as add numbers. The simplest way of doing this involves writing the complement of a number, and then complementing the final result with the complement.

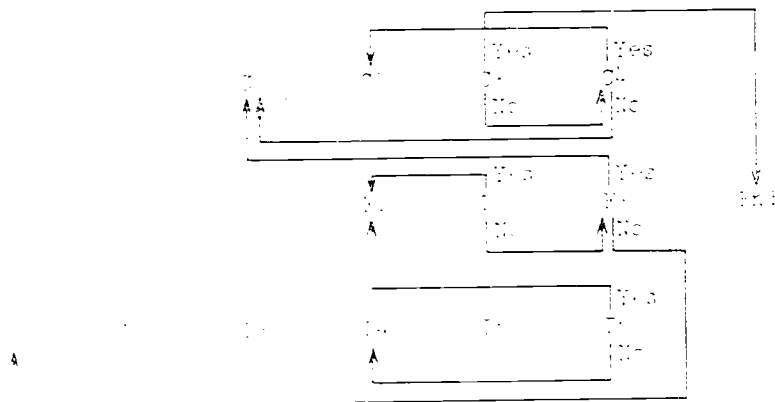
Activity 1. The Prime Sieve (Part I)

Object: To demonstrate the use of storing to find the prime numbers, and to illustrate the use of a procedure.

Participants: Seven students, called, respectively, Isaac, Paul, Gepp, Alan, James, Robert, and Harriet.

Procedure: The program, presented, alternates two subroutines until a program Find is found where $N = 101$ for the example. Then the third step, the routine, is executed, resulting primes not greater than N . The participants Isaac, James, and Gepp record the sequence of steps in their respective subroutines, and the other four students perform specific functions. The program is compared with the numbers 1 to N written on the blackboard, and the Delete step is executed: the largest prime determined to be removed from the board, a number is multiplied starting with the square of that prime. The program is found, and this agreement works properly. The Find step is then executed, and the next larger prime to be located, and if this prime is not found, Find, it returns control to Delete; otherwise control passes to Copy, which records the remaining primes to be listed directly.

Summary: The participants find, testing 1 as the first prime, and this is recorded in Copy. Then Delete starts its subroutine. The sequencing of the program is shown in the following diagram, in which the steps of each subroutine are indicated by separate lines:



These steps are described in the following table:

| <u>STEP</u> | <u>PARADIGM</u> | <u>ACTION</u> |
|-------------|-----------------|---|
| 11 | <u>List</u> | Enters the number held by <u>Next</u> on the list of primes. |
| 12 | <u>Erase</u> | Erases from the table the number held by <u>Next</u> . |
| 13 | <u>Number</u> | Calculates the square of the number held by <u>Next</u> . |
| 14 | <u>Erase</u> | Erases from the table the number held by <u>Number</u> . |
| 15 | <u>Number</u> | Adds to his number the number held by <u>Next</u> . |
| 16 | <u>Number</u> | Calls "Yes" if his sum is greater than $\sqrt{100}$, otherwise calls "No." |
| 17 | <u>Next</u> | Adds 1 to the number he holds. |
| 18 | <u>Next</u> | Calls "Yes" if the number he now holds has been erased, otherwise "No." |
| 19 | <u>Next</u> | Calls "Yes" if the number he holds is greater than 10, otherwise "No." |
| 20 | <u>List</u> | Enters the number held by <u>Next</u> on the list of primes. |
| 21 | <u>Next</u> | Adds 2 to the number he holds. |
| 22 | <u>Next</u> | Calls "Yes" if the number exceeds 100, otherwise "No." |
| 23 | <u>Next</u> | Calls "Yes" if the number has been erased, otherwise "No." |

Find initiates his subroutine when "Yes" is called on Step 16. Delete returns his when Next calls "No" on Step 18; if instead he calls "Yes," Copy initiates his subroutine. At this point, all primes $\leq \sqrt{100}$ have been listed, and the numbers in the table that have not been erased are all primes. (The reader should find the reason for this rule.)

It will often happen that Erase is called on to remove from the table a number that has already been erased. When will this occur? In this case he does nothing. It can be pointed out in discussion that erasing a blank leaves a blank, and that making use of this fact simplifies the program; otherwise the table would have to be examined before deciding to erase. Also, the need for a subroutine Top should be discussed for its insight into the storage of information and into the numbers that remain when this subroutine begins are irrelevant. Certainly, some method of organizing them is required. Visual scanning is a technique that its convenience tends to be overlooked.

W. J. G. .

JUMPING GAME



Five (the number is optional) boards 3 feet by 10 inches are joined together so that they will not separate when children jump upon them. The object of the game is to jump onto the blue (last) board successfully. A player is not successful should he land on a line or should he copy a trip made by a fellow player. A trip may consist of any number of jumps of any length as long as the jumps proceed toward the blue board. That is a player may not jump back toward the start. The trips always start on the floor and end up on the blue board. Also, a player must commit himself to a specific trip, before he starts his jump, by recording his proposed trip on the chart. If he completes his trip successfully he enters his name along side of his proposed trip. An unsuccessful player may try his trip again on his next turn unless someone else has taken it successfully. If either of these situations arise he may try a new trip if he can think of one that has not been used up. For example, suppose a child proposed himself to jumping directly on the yellow and then on the blue, and did it on the chart. Thus, all of the trips that have been used up are marked on the chart.

The player with the most successes wins the game. This game should be played by a minimum of 4 players. The winning player starts last for the next game. Boards may be added or taken away depending upon the ability of the players. This game is useful for developing systems for exhausting all possibilities as well as giving very young students experience in extending their knowledge from two, three, and so forth by such activities as:

If you start on the orange board and take a jump of two, where will you land?

If you start on the red board and land on the green board, how long was your jump?

If you land on the blue board by taking a jump of 3, where do you start?

How many jumps of 3 would you need to take to get you as far as a jump of 6?

If you need a jump of 3 and then a jump of 3, how far would your friend have to jump to get to where you are in the jump?

etc.

The activity also motivates the interesting problem:
How many trips are possible using, say, 7 boards?

This problem lends itself to several approaches depending upon the level of the student to whom it is presented. To illustrate a few of the possibilities we will consider approaches appropriate for elementary students, junior-high students, and high school students, respectively.

- Suppose I had just one board. How many trips are open to me? For two boards? Three? Let's list our results:

| | | | | | | |
|--------|---|---|---|---|---|-----|
| Boards | 1 | 2 | 3 | 4 | 5 | ... |
| Trips | 1 | 2 | 3 | 4 | 5 | ... |

Suppose I have played the game with 4 boards and find that I have 4 possible trips. Now I add one more board. Consider each of the possible trips in the 4-board game. Each of these trips ended by jumping onto the blue board, say. But now instead of jumping on the blue board, I have two choices. I can jump on the blue board or directly onto the new last board. Hence, I have twice as many possible trips as before. This gives me a recursion formula for determining the number of possible trips since I know that with 1 board I can only 1 trip.

$$\begin{array}{c}
 p(1) = 1 \\
 \text{and} \\
 \forall i, p(i+1) = i + p(i)
 \end{array}$$

- 4) For those elements who have discovered and verified that the number of subsets of an n -element set is 2^n , another solution is available. Since each trip onto the blue board, the set of boards jumped upon before landing upon the blue board determines a trip. For example, what trip corresponds to the empty set of boards? Hence, the problem simply boils down to, "How many different sets of non-blue boards are there?" This of course is the number of subsets of a set consisting of $k-1$ elements where k is the total number of boards including the blue board.

H. Miller

ARITHMETIC SEQUENCES

Many teachers are aware of the fact that children like to puzzle out the "next" term of a sequence after having been given several preceding ones. In a sequence, however, the students are left with the impressions listed below.

1. There is no dependence with the terms given.
2. There must always be a next term.
3. Sequences must be a terminating sequence "tolerant" rule.

It is unfortunate that such a pleasant positive results in some unhelpful concepts. In order to obtain these ideas and at the same time eliminate non-terminating sequences, the following instructions are made.

Present a sequence of terms like:

$$1, 4, 9, 16, 25, 36$$

and ask students for the next term of a sequence which has the beginning shown above. After they give their next term they are to give some kind of rule for generating the whole sequence. This approach promotes creativity and graphically illustrates the problem for the next term of an unspecified sequence is inappropriate. This activity could be started with some given terms and gradually worked up until the students give their definition.

For instance, the child starts by giving every two as the first term. Then at least three or four:

$$2, 4$$

then:

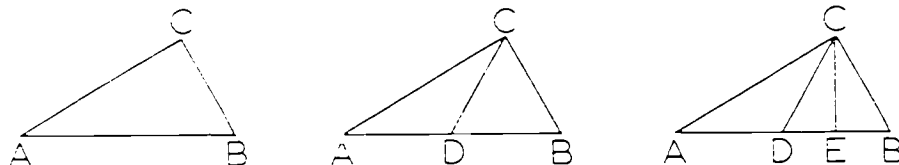
$$2, 4, 6$$

Sequences which start with:

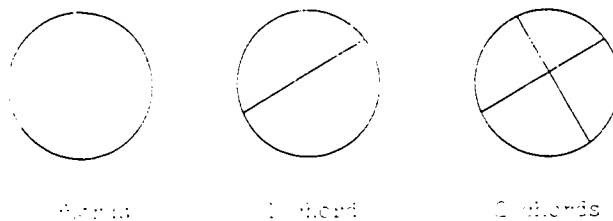
$$1, 2, 3, 4, 5, 6$$

may be especially interesting since they are sequences which are common to the student's world. This way of presenting sequences varies from the given set of two sequences to various others. Here are a few examples:

Another drawing line segment r from the vertex of a triangle to its base. With the same line segments drawn you have only one n -part area. With one extra line segment drawn you have two non-overlapping triangles pictured. Now, suppose w is really fast on the ball and conjecture that drawing will result in four parts with two line segments. He will have to have non-overlapping regions. This is not the case as is illustrated below.

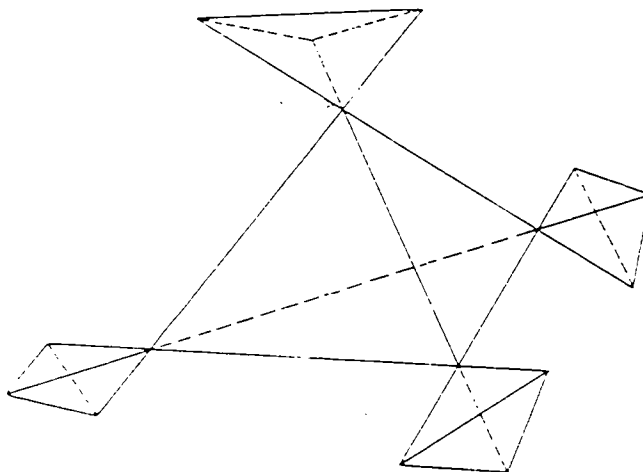


Let us now consider a situation which will generate a sequence which starts out $1, 2, 3, 4, \dots$. Here, chords in a circle is said to be that the maximum number of regions are formed:



Let us now ask the question: if chords we get a maximum of n regions.

It is a frequent conjecture about a geometric situation yielding a sequence which starts out $1, 2, 3, 4, \dots$. The question is, how many 3-dimensional regions are formed by n mutually intersecting planes. Three planes, one plane, two planes, and three planes yield $1, 2, 3, 4, \dots$ regions respectively. He furnishes a diagram of a drawing for the case of 4 planes in space:



$$\begin{array}{rcl}
1 & + & 1 = 2 \\
1 & + & 3 = 4 \\
1 & + & 5 = 6 \\
1 & + & 7 = 8 \\
1 & + & 9 = 10 \\
1 & + & 11 = 12 \\
1 & + & 13 = 14 \\
1 & + & 15 = 16 \\
1 & + & 17 = 18 \\
1 & + & 19 = 20 \\
1 & + & 21 = 22 \\
1 & + & 23 = 24 \\
1 & + & 25 = 26 \\
1 & + & 27 = 28 \\
1 & + & 29 = 30 \\
1 & + & 31 = 32
\end{array}$$

There may be an interesting possibility. If the numbers listed are multiplied, the results are:

$$\frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{1}{6} \cdot \frac{1}{7} \cdot \frac{1}{8} \cdot \frac{1}{9} \cdot \frac{1}{10} \cdot \frac{1}{11} \cdot \frac{1}{12} \cdot \frac{1}{13} \cdot \frac{1}{14} \cdot \frac{1}{15} \cdot \frac{1}{16} \cdot \frac{1}{17} \cdot \frac{1}{18} \cdot \frac{1}{19} \cdot \frac{1}{20} \cdot \frac{1}{21} \cdot \frac{1}{22} \cdot \frac{1}{23} \cdot \frac{1}{24} \cdot \frac{1}{25} \cdot \frac{1}{26} \cdot \frac{1}{27} \cdot \frac{1}{28} \cdot \frac{1}{29} \cdot \frac{1}{30} \cdot \frac{1}{31} \cdot \frac{1}{32} = \frac{1}{32!}$$

It would also be interesting to construct sequences from recursive rules which yield other results. We have several examples of these in the EMIS-ITEMA "Gold Mine".

Recursive sequences may be used to familiarize students with certain patterns which come up again and again in mathematics. For instance we have a long list of recursive sequences where the powers-of-two pattern comes up in the Fibonacci sequence, the Tower of Hanoi, and the Trains of Pods. A sequence of recursive sequences may be used in the classroom activity when it precedes a topic involving the same or related pattern. They may also be used immediately following the discovery of a pattern so as to give students more experience with it. This is important for kids since they could not easily discover a pattern which they were unfamiliar. To make this point clearer, consider a class which is working on squaring numbers. When these students are faced with numbers such as 144 and 169, they immediately recognize them as squares. Consider another class without this experience. Now face them with questions such as:

What is the sum of the first 10 odd numbers?

What is the sum of the first 100 odd numbers?

What is the sum of the first 1000 odd numbers? etc.

It would be a good question for the class with the experience described above. It would be a harder question for the class without the experience. It would be a good question for the class with the experience described above.

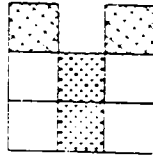
What is the sum of the first 1000 odd numbers?

What is the sum of the first 10000 odd numbers?

A. White

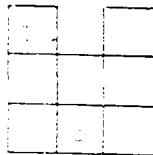
THE CODE P "Y"

A. The student is given a grid with numbers already entered in some of the squares. The student is to enter the numbers 1 through 9 in the remaining squares so that each row, column, and 3x3 box contains the numbers 1 through 9.

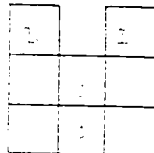


B. The student is given a "Y" shape of the arrangement of the shaded squares. The student is to enter the numbers 1 through 9 in the squares of the "Y". The student is to enter the numbers 1 through 9 in the squares of the "Y" so that each row, column, and 3x3 box contains the numbers 1 through 9. The student is to enter the numbers 1 through 9 in the squares of the "Y" so that each row, column, and 3x3 box contains the numbers 1 through 9. The student is to enter the numbers 1 through 9 in the squares of the "Y" so that each row, column, and 3x3 box contains the numbers 1 through 9.

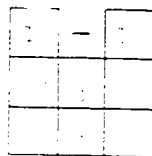
C. The student is given a grid with numbers already entered in some of the squares. The student is to enter the numbers 1 through 9 in the remaining squares so that each row, column, and 3x3 box contains the numbers 1 through 9.



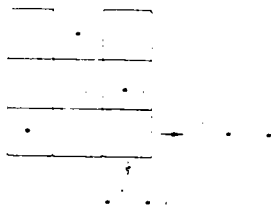
D. The student is given a grid with numbers already entered in some of the squares. The student is to enter the numbers 1 through 9 in the remaining squares so that each row, column, and 3x3 box contains the numbers 1 through 9.



E. The student is given a grid with numbers already entered in some of the squares. The student is to enter the numbers 1 through 9 in the remaining squares so that each row, column, and 3x3 box contains the numbers 1 through 9. The student is to enter the numbers 1 through 9 in the squares of the "Y" so that each row, column, and 3x3 box contains the numbers 1 through 9. The student is to enter the numbers 1 through 9 in the squares of the "Y" so that each row, column, and 3x3 box contains the numbers 1 through 9.



In Figure 10, the two lines are drawn with the same slope, but the line on the right is shifted. This illustrates how the slope of a line is determined.



When a line is drawn with a slope of m , the line is parallel to the line with slope m . This is because the slope of a line is a measure of its steepness. If two lines have the same slope, they are parallel. This is true for any two lines in a plane. If two lines are not parallel, they will intersect at exactly one point. If two lines are parallel, they will never intersect. This is a fundamental property of lines in a plane.

• When $m = 0$, the line is horizontal.

When m is a negative number, the line has a negative slope. This means that as x increases, y decreases.

When m is a positive number, the line has a positive slope. This means that as x increases, y increases.

When m is a negative number, the line has a negative slope. This means that as x increases, y decreases.

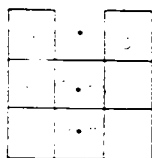
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$$y - y_1 = m(x - x_1)$$

In "Y" form, the equation of a line is expressed as $y = mx + b$. In this form, m is the slope of the line and b is the y-intercept.



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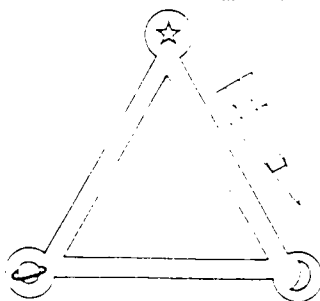
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For the first time, the children had a chance to play independently. The teacher was a passive participant in the play. In a different situation, the children had been playing with the following:



From this, the children were a number of days away from the fact that the teacher was a passive participant in the play. For the first time, the children had a chance to play independently. The teacher was a passive participant in the play. In a different situation, the children had been playing with the following:



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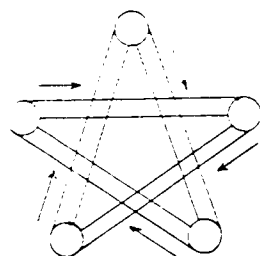
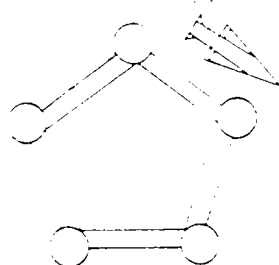
The children were a number of days away from the fact that the teacher was a passive participant in the play. For the first time, the children had a chance to play independently. The teacher was a passive participant in the play. In a different situation, the children had been playing with the following:

$$1. \quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

The children were a number of days away from the fact that the teacher was a passive participant in the play. For the first time, the children had a chance to play independently. The teacher was a passive participant in the play. In a different situation, the children had been playing with the following:

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$$x \xrightarrow{A} y$$

...the ...
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...the ...

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...the ...
...the ...

H. Miller

1. The first step is to identify the key components of the system. This includes understanding the hardware, software, and data involved.

[illegible][illegible][illegible]

R. F. DODGE
R. F. DODGE

THE BIRTH OF THE NATION

...and the first of the great men of the nation were born. ...and the first of the great men of the nation were born.

...and the first of the great men of the nation were born. ...and the first of the great men of the nation were born.

...and the first of the great men of the nation were born. ...and the first of the great men of the nation were born.

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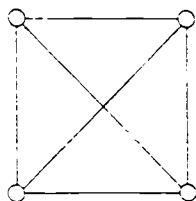
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CONSTRUCTIONS

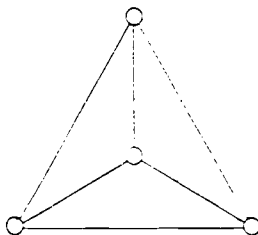
Constructing figures can be useful in developing spatial imagination. The points in the constructions below were not all justified, but some of them could easily be justified. The first five have a high-quality quality. The first sets on two elements, the patterns are plane-like figures. The second set on the fourth or fifth element, the patterns are three-dimensional, parallel for the third set.

The first construction, the one, is considered as justified, but it is not. The second, described with words, it is not, but it is a possible diagram, as in this example, which does not seem to be a paper and pencil exercise for children, but a mental exercise for older children.

Constructing figures is similar to the elements so that every pair is connected. It is not a difficult task. For example, make the points so that no two of the points are collinear. That people visualize this:



Now, make the points so that no three are collinear, and the points are:



Now, make the points so that no three are collinear, and the points are: no three are collinear. It is required, of course, that the points are distinct, and that no three points are collinear.

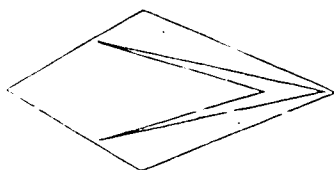
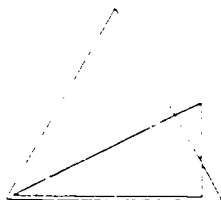
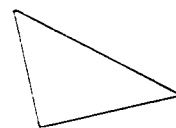


Figure 1, which will be quite new, was proposed by a Russian mathematician, Vladimir Alexandrovich. It is a parallelogram with the center of a circle and two tangents to it at the circle's main diameter. In what follows, the side of the parallelogram, AB , will be denoted as a sufficiently vivid mental image. In the diagram, the circle is depicted in gray. In working on other problems, one often resorts to the use of this parallelogram as a kind of standard weapon. They arrive at this by means of the properties of the figure. Three of its vertices have a common center of rotation, and the other two have a common vertex at the center of the circle. This aids one in one's visualization. Since the parallelogram is also a rectangle, it is perfectly clear that it is equally symmetric with respect to the diameter AB and CD . Also, by symmetry, the line segments AD and BC are equal in length. This is not a proof of a new theorem, but it is a figure which is useful to us in our work.

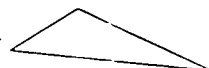
4. Copy the triangle onto the grid. If this side has a length of 3 units, then the other two sides are 4 units and 5 units.



5. Copy the triangle.

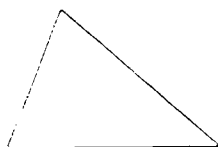


6. Copy the triangle.



7. Draw a line segment of length 3 units. Then draw a line segment of length 4 units perpendicular to it.

8. Connect the endpoints of the two line segments.



9. Draw a line segment of length 3 units. Then draw a line segment of length 4 units perpendicular to it.



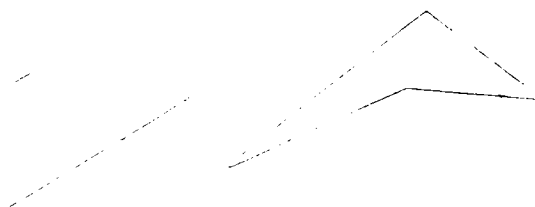
10. Draw a line segment of length 3 units. Then draw a line segment of length 4 units perpendicular to it.



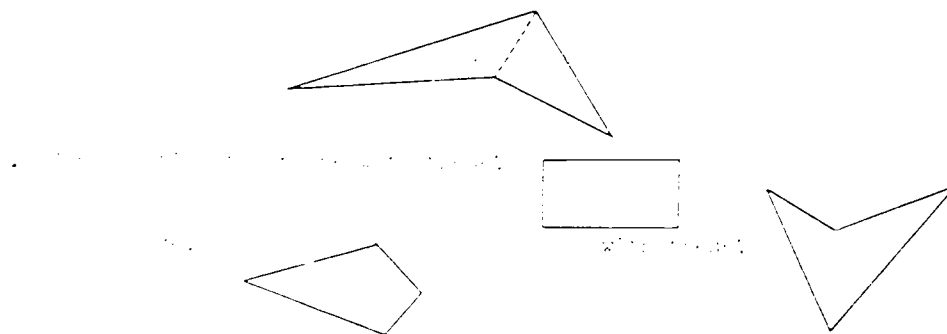
11. Draw a line segment of length 3 units. Then draw a line segment of length 4 units perpendicular to it. Connect the endpoints of the two line segments. This is the triangle you are looking for. It is a right-angled triangle with a horizontal base of 3 units and a vertical height of 4 units. The hypotenuse is 5 units long.



12. Draw a line segment of length 4 units perpendicular to the first line segment.

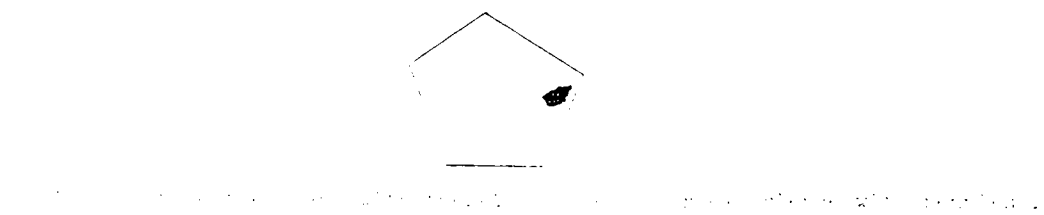


at 2 p.p.m. - 4 p.m. (1 hour) for the first day, and then to 1 p.m. - 4 p.m. for the second day. This will be the full complement. Start with the same complement the following day.

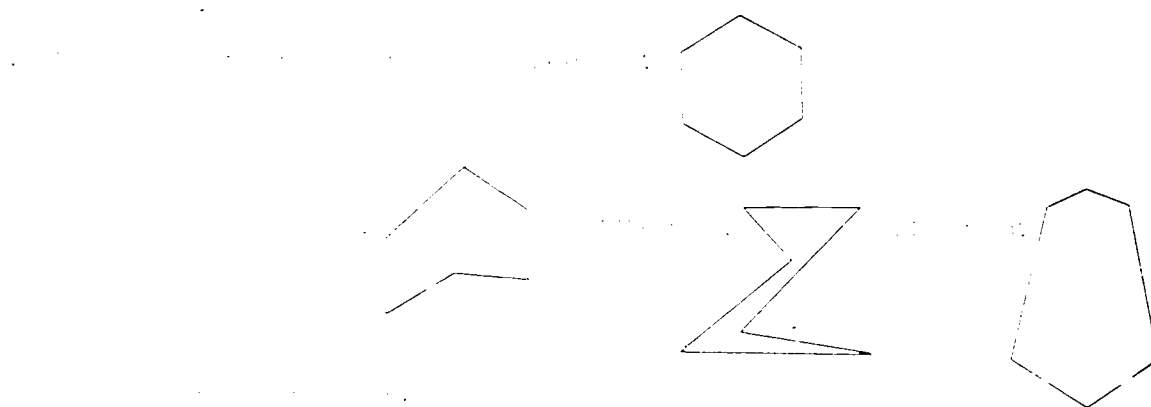


• The following are the shapes that will be used for the second day:

• The following are the shapes that will be used:



• The following are the shapes that will be used for the third day:



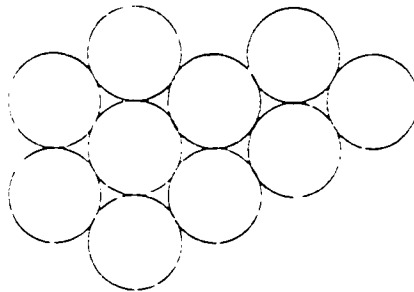
• The following are the shapes that will be used for the fourth day:

ACTIVITY

HEXAGY-PACKING

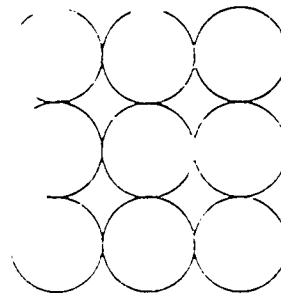
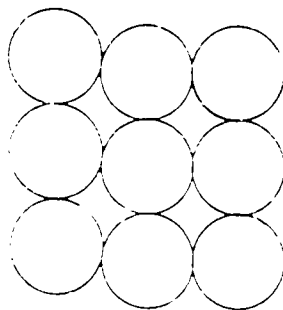
1. 100

- 1. 100. The cluster structure is possible:



2. 100

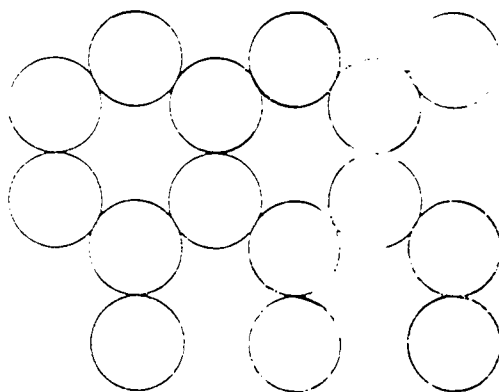
3. 100



4. 100

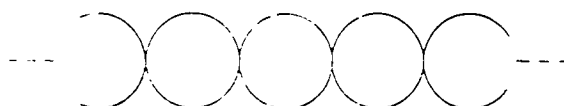
5. 100

Fig. 2.

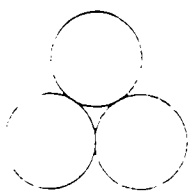


- each circle group is surrounded by 6 circles;
- each circle group is surrounded by 5 circles.

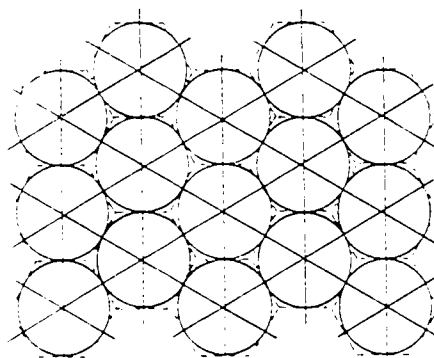
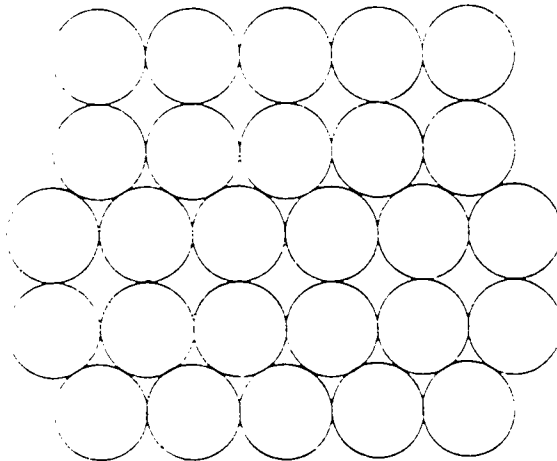
Fig. 3. (a) - packing; (b) - packing.



The number of circles in the cluster is infinite in all directions. The number of circles in the cluster is infinite in all directions. The number of circles in the cluster is infinite in all directions.



1. The solid lines represent the atoms.
 2. The dashed lines represent the bonds.



Solid lines join centers
 of circles; dashed lines
 join centers of spheres.
 (Dashed lines represent the
 "bonds".)
 Solid lines are solid lines
 and solid lines.
 Dashed lines are dashed
 lines and dashed lines.

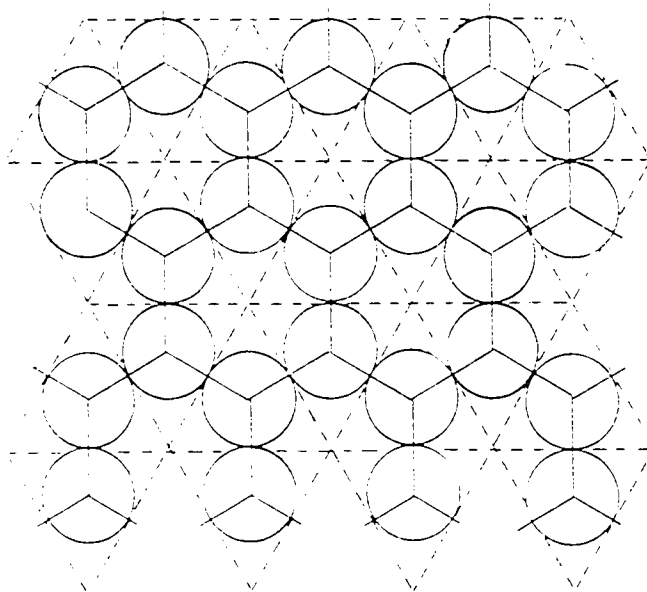


Fig. 1. The pattern of the circles and the dashed lines are the same as in the pattern of the circles. The dashed lines form triangular cells; the circles form hexagonal cells.

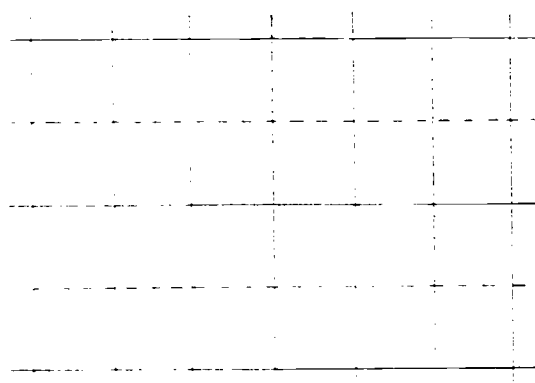


Fig. 2. The pattern of the dashed lines and the solid lines are the same as in the pattern of the circles. The dashed lines form triangular cells; the solid lines form hexagonal cells.

- The relative velocity is that of moving the balls along in x , or the x -axis in Fig. 1-10.
- It is not a simple problem without it.

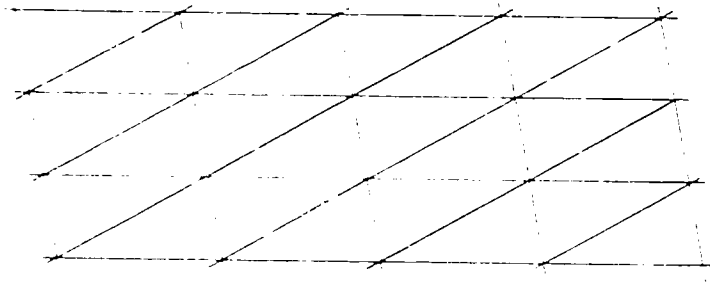


Fig. 1-10. World lines.

Fig. 1-11. World lines in a frame of reference S .

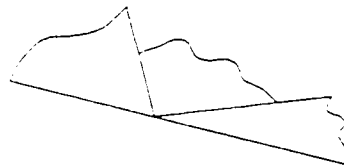


Fig. 1-12. World lines in a frame of reference S . The world lines A, B , and C are parallel to each other, but are not parallel to the x -axis.

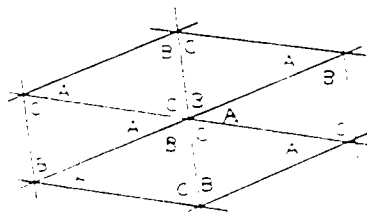
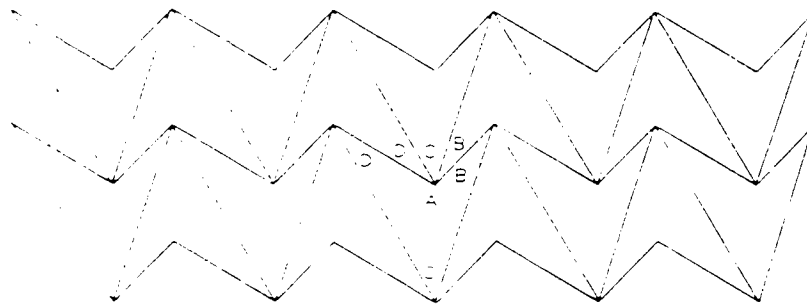
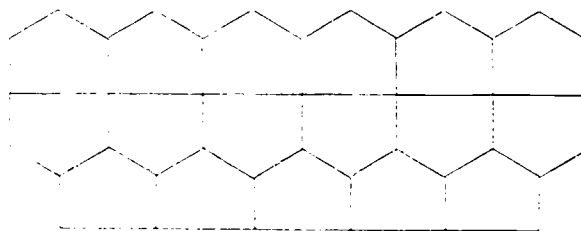


Fig. 1-13. World lines in a frame of reference S . The world lines A, B , and C are parallel to each other, but are not parallel to the x -axis.

- A 100 ft. wide strip of material will fill a 100 ft. wide ditch. Divide the 100 ft. wide strip into 10 equal parts. The surface of each strip will be 100 ft. wide, 100 ft. high, and 100 ft. long. A 100 ft. wide strip will fill a 100 ft. wide ditch.
- A 100 ft. wide strip of material will fill a 100 ft. wide ditch. The surface of each strip will be 100 ft. wide, 100 ft. high, and 100 ft. long. A 100 ft. wide strip will fill a 100 ft. wide ditch.



- A 100 ft. wide strip of material will fill a 100 ft. wide ditch. The surface of each strip will be 100 ft. wide, 100 ft. high, and 100 ft. long. A 100 ft. wide strip will fill a 100 ft. wide ditch.
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- A 100 ft. wide strip of material will fill a 100 ft. wide ditch. The surface of each strip will be 100 ft. wide, 100 ft. high, and 100 ft. long. A 100 ft. wide strip will fill a 100 ft. wide ditch.

[illegible]

1. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Arar and Collins (1971).
 2. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Arar and Collins (1971).



- If \mathcal{H} is a Hilbert space and \mathcal{H}^* is the dual space of \mathcal{H} , then \mathcal{H}^{**} is isomorphic to \mathcal{H} . The canonical map is $\mathcal{H} \rightarrow \mathcal{H}^{**}$.
- If \mathcal{H} is a Hilbert space and \mathcal{H}^* is the dual space of \mathcal{H} , then \mathcal{H}^{**} is isomorphic to \mathcal{H} .
- If \mathcal{H} is a Hilbert space and \mathcal{H}^* is the dual space of \mathcal{H} , then \mathcal{H}^{**} is isomorphic to \mathcal{H} .
- If \mathcal{H} is a Hilbert space and \mathcal{H}^* is the dual space of \mathcal{H} , then \mathcal{H}^{**} is isomorphic to \mathcal{H} .

ANSWERS

MINI-BOOKING

1. No. It is certainly true that two squares share the same center just like two regular hexagons.
2. Twelve. Four in the outer layer, four more in the center.
3. No. The regular polyhedron would have to be an icosahedron, which has 20 triangular faces. But the polyhedron with vertices at the centers of the faces has two square faces. The four points of contact with the upper sphere, the midpoints, are vertices of a square face.
4. It is possible.
5. No. The 12 faces are squares, 8 above, 4 below.
6. It is close to square faces, though they are a little larger than the other faces. Alternatively, observe that the polyhedron is symmetric with respect to the plane through the six contact points in the sphere's center. Therefore, by symmetry an even number of faces must come together at each of these six points. But the icosahedron has an odd number of faces at each vertex.
7. No.
8. No.
9. No.

L. W. Brown

the following is a list of the

names of the persons who have been
admitted to the office of the
Recorder of Deeds for the year
1900. The names are given in
alphabetical order of the surnames.

— A —

Adams, John, 1000 1/2
Adams, John, 1000 1/2

— B —

Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2

Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2
Baker, John, 1000 1/2

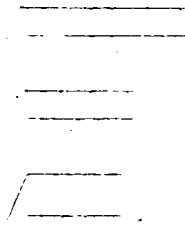
— C —

Campbell, John, 1000 1/2
Campbell, John, 1000 1/2
Campbell, John, 1000 1/2

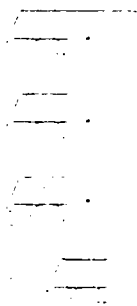
5-1-15



1. *Phragmites australis* (Cav.) Trin. ex Steud.



The first of these is the fact that the high
 school is a very important part of the
 community. It is a place where the young
 people of the community are educated and
 where they learn to live together in a
 democratic society.



The second of these is the fact that the high
 school is a place where the young people of
 the community are educated and where they
 learn to live together in a democratic society.
 The third of these is the fact that the high
 school is a place where the young people of
 the community are educated and where they
 learn to live together in a democratic society.



The fourth of these is the fact that the high

school

100

[illegible]

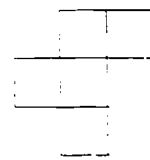
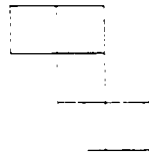
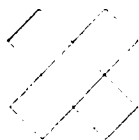
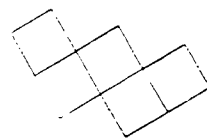
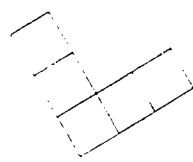
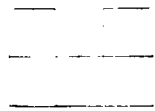
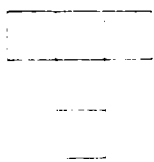
| Age Group | 2006 | 2007 | 2008 |
|-----------|------|------|------|
| 18-29 | ~85 | ~85 | ~85 |
| 30-49 | ~75 | ~75 | ~75 |
| 50-69 | ~65 | ~65 | ~65 |
| 70+ | ~55 | ~55 | ~55 |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|

the 1990s, the number of people in the world who are illiterate has increased from 1.2 billion to 1.5 billion. The number of illiterate people in the world is projected to reach 1.7 billion by the year 2015. The number of illiterate people in the world is projected to reach 1.7 billion by the year 2015.

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185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 | 225 | 226 | 227 | 228 | 229 | 230 | 231 | 232 | 233 | 234 | 235 | 236 | 237 | 238 | 239 | 240 | 241 | 242 | 243 | 244 | 245 | 246 | 247 | 248 | 249 | 250 | 251 | 252 | 253 | 254 | 255 | 256 | 257 | 258 | 259 | 260 | 261 | 262 | 263 | 264 | 265 | 266 | 267 | 268 | 269 | 270 | 271 | 272 | 273 | 274 | 275 | 276 | 277 | 278 | 279 | 280 | 281 | 282 | 283 | 284 | 285 | 286 | 287 | 288 | 289 | 290 | 291 | 292 | 293 | 294 | 295 | 296 | 297 | 298 | 299 | 300 | 301 | 302 | 303 | 304 | 305 | 306 | 307 | 308 | 309 | 310 | 311 | 312 | 313 | 314 | 315 | 316 | 317 | 318 | 319 | 320 | 321 | 322 | 323 | 324 | 325 | 326 | 327 | 328 | 329 | 330 | 331 | 332 | 333 | 334 | 335 | 336 | 337 | 338 | 339 | 340 | 341 | 342 | 343 | 344 | 345 | 346 | 347 | 348 | 349 | 350 | 351 | 352 | 353 | 354 | 355 | 356 | 357 | 358 | 359 | 360 | 361 | 362 | 363 | 364 | 365 | 366 | 367 | 368 | 369 | 370 | 371 | 372 | 373 | 374 | 375 | 376 | 377 | 378 | 379 | 380 | 381 | 382 | 383 | 384 | 385 | 386 | 387 | 388 | 389 | 390 | 391 | 392 | 393 | 394 | 395 | 396 | 397 | 398 | 399 | 400 | 401 | 402 | 403 | 404 | 405 | 406 | 407 | 408 | 409 | 410 | 411 | 412 | 413 | 414 | 415 | 416 | 417 | 418 | 419 | 420 | 421 | 422 | 423 | 424 | 425 | 426 | 427 | 428 | 429 | 430 | 431 | 432 | 433 | 434 | 435 | 436 | 437 | 438 | 439 | 440 | 441 | 442 | 443 | 444 | 445 | 446 | 447 | 448 | 449 | 450 | 451 | 452 | 453 | 454 | 455 | 456 | 457 | 458 | 459 | 460 | 461 | 462 | 463 | 464 | 465 | 466 | 467 | 468 | 469 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 477 | 478 | 479 | 480 | 481 | 482 | 483 | 484 | 485 | 486 | 487 | 488 | 489 | 490 | 491 | 492 | 493 | 494 | 495 | 496 | 497 | 498 | 499 | 500 | 501 | 502 | 503 | 504 | 505 | 506 | 507 | 508 | 509 | 510 | 511 | 512 | 513 | 514 | 515 | 516 | 517 | 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851 | 852 | 853 | 854 | 855 | 856 | 857 | 858 | 859 | 860 | 861 | 862 | 863 | 864 | 865 | 866 | 867 | 868 | 869 | 870 | 871 | 872 | 873 | 874 | 875 | 876 | 877 | 878 | 879 | 880 | 881 | 882 | 883 | 884 | 885 | 886 | 887 | 888 | 889 | 890 | 891 | 892 | 893 | 894 | 895 | 896 | 897 | 898 | 899 | 900 | 901 | 902 | 903 | 904 | 905 | 906 | 907 | 908 | 909 | 910 | 911 | 912 | 913 | 914 | 915 | 916 | 917 | 918 | 919 | 920 | 921 | 922 | 923 | 924 | 925 | 926 | 927 | 928 | 929 | 930 | 931 | 932 | 933 | 934 | 935 | 936 | 937 | 938 | 939 | 940 | 941 | 942 | 943 | 944 | 945 | 946 | 947 | 948 | 949 | 950 | 951 | 952 | 953 | 954 | 955 | 956 | 957 | 958 | 959 | 960 | 961 | 962 | 963 | 964 | 965 | 966 | 967 | 968 | 969 | 970 | 971 | 972 | 973 | 974 | 975 | 976 | 977 | 978 | 979 | 980 | 981 | 982 | 983 | 984 | 985 | 986 | 987 | 988 | 989 | 990 | 991 | 992 | 993 | 994 | 995 | 996 | 997 | 998 | 999 | 1000 | 1001 | 1002 | 1003 | 1004 | 1005 | 1006 | 1007 | 1008 | 1009 | 1010 | 1011 | 1012 | 1013 | 1014 | 1015 | 1016 | 1017 | 1018 | 1019 | 1020 | 1021 | 1022 | 1023 | 1024 | 1025 | 1026 | 1027 | 1028 | 1029 | 1030 | 1031 | 1032 | 1033 | 1034 | 1035 | 1036 | 1037 | 1038 | 1039 | 1040 | 1041 | 1042 | 1043 | 1044 | 1045 | 1046 | 1047 | 1048 | 1049 | 1050 | 1051 | 1052 | 1053 | 1054 | 1055 | 1056 | 1057 | 1058 | 1059 | 1060 | 1061 | 1062 | 1063 | 1064 | 1065 | 1066 | 1067 | 1068 | 1069 | 1070 | 1071 | 1072 | 1073 | 1074 | 1075 | 1076 | 1077 | 1078 | 1079 | 1080 | 1081 | 1082 | 1083 | 1084 | 1085 | 1086 | 1087 | 1088 | 1089 | 1090 | 1091 | 1092 | 1093 | 1094 | 1095 | 1096 | 1097 | 1098 | 1099 | 1100 | 1101 | 1102 | 1103 | 1104 | 1105 | 1106 | 1107 | 1108 | 1109 | 1110 | 1111 | 1112 | 1113 | 1114 | 1115 | 1116 | 1117 | 1118 | 1119 | 1120 | 1121 | 1122 | 1123 | 1124 | 1125 | 1126 | 1127 | 1128 | 1129 | 1130 | 1131 | 1132 | 1133 | 1134 | 1135 | 1136 | 1137 | 1138 | 1139 | 1140 | 1141 | 1142 | 1143 | 1144 | 1145 | 1146 | 1147 | 1148 | 1149 | 1150 | 1151 | 1152 | 1153 | 1154 | 1155 | 1156 | 1157 | 1158 | 1159 | 1160 | 1161 | 1162 | 1163 | 1164 | 1165 | 1166 | 1167 | 1168 | 1169 | 1170 | 1171 | 1172 | 1173 | 1174 | 1175 | 1176 | 1177 | 1178 | 1179 | 1180 | 1181 | 1182 | 1183 | 1184 | 1185 | 1186 | 1187 | 1188 | 1189 | 1190 | 1191 | 1192 | 1193 | 1194 | 1195 | 1196 | 1197 | 1198 | 1199 | 1200 | 1201 | 1202 | 1203 | 1204 | 1205 | 1206 | 1207 | 1208 | 1209 | 1210 | 1211 | 1212 | 1213 | 1214 | 1215 | 1216 | 1217 | 1218 | 1219 | 1220 | 1221 | 1222 | 1223 | 1224 | 1225 | 1226 | 1227 | 1228 | 1229 | 1230 | 1231 | 1232 | 1233 | 1234 | 1235 | 1236 | 1237 | 1238 | 1239 | 1240 | 1241 | 1242 | 1243 | 1244 | 1245 | 1246 | 1247 | 1248 | 1249 | 1250 | 1251 | 1252 | 1253 | 1254 | 1255 | 1256 | 1257 | 1258 | 1259 | 1260 | 1261 | 1262 | 1263 | 1264 | 1265 | 1266 | 1267 | 1268 | 1269 | 1270 | 1271 | 1272 | 1273 | 1274 | 1275 | 1276 | 1277 | 1278 | 1279 | 1280 | 1281 | 1282 | 1283 | 1284 | 1285 | 1286 | 1287 | 1288 | 1289 | 1290 | 1291 | 1292 | 1293 | 1294 | 1295 | 1296 | 1297 | 1298 | 1299 | 1300 | 1301 | 1302 | 1303 | 1304 | 1305 | 1306 | 1307 | 1308 | 1309 | 1310 | 1311 | 1312 | 1313 | 1314 | 1315 | 1316 | 1317 | 1318 | 1319 | 1320 | 1321 | 1322 | 1323 | 1324 | 1325 | 1326 | 1327 | 1328 | 1329 | 1330 | 1331 | 1332 | 1333 | 1334 | 1335 | 1336 | 1337 | 1338 | 1339 | 1340 | 1341 | 1342 | 1343 | 1344 | 1345 | 1346 | 1347 | 1348 | 1349 | 1350 | 1351 | 1352 | 1353 | 1354 | 1355 | 1356 | 1357 | 1358 | 1359 | 1360 | 1361 | 1362 | 1363 | 1364 | 1365 | 1366 | 1367 | 1368 | 1369 | 1370 | 1371 | 1372 | 1373 | 1374 | 1375 | 1376 | 1377 | 1378 | 1379 | 1380 | 1381 | 1382 | 1383 | 1384 | 1385 | 1386 | 1387 | 1388 | 1389 | 1390 | 1391 | 1392 | 1393 | 1394 | 1395 | 1396 | 1397 | 1398 | 1399 | 1400 | 1401 | 1402 | 1403 | 1404 | 1405 | 1406 | 1407 | 1408 | 1409 | 1410 | 1411 | 1412 | 1413 | 1414 | 1415 | 1416 | 1417 | 1418 | 1419 | 1420 | 1421 | 1422 | 1423 | 1424 | 1425 | 1426 | 1427 | 1428 | 1429 | 1430 | 1431 | 1432 | 1433 | 1434 | 1435 | 1436 | 1437 | 1438 | 1439 | 1440 | 1441 | 1442 | 1443 | 1444 | 1445 | 1446 | 1447 | 1448 | 1449 | 1450 | 1451 | 1452 | 1453 | 1454 | 1455 | 1456 | 1457 | 1458 | 1459 | 1460 | 1461 | 1462 | 1463 | 1464 | 1465 | 1466 | 1467 | 1468 | 1469 | 1470 | 1471 | 1472 | 1473 | 1474 | 1475 | 1476 | 1477 | 1478 | 1479 | 1480 | 1481 | 1482 | 1483 | 1484 | 1485 | 1486 | 1487 | 1488 | 1489 | 1490 | 1491 | 1492 | 1493 | 1494 | 1495 | 1 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-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The first of these is the fact that the number of people who are employed in the service industry has increased steadily over the past few years. This is due to a number of factors, including the fact that the service industry is becoming more important in the economy as a whole. Another factor is the fact that the service industry is becoming more important in the lives of individuals as well. For example, the service industry is becoming more important in the lives of individuals as they age, as they need more services than they did when they were younger. This is also true for people who are disabled or who have other special needs. The service industry is becoming more important in the lives of individuals as they age, as they need more services than they did when they were younger. This is also true for people who are disabled or who have other special needs.

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The second of these factors is the fact that the service industry is becoming more important in the lives of individuals as they age, as they need more services than they did when they were younger. This is also true for people who are disabled or who have other special needs. The service industry is becoming more important in the lives of individuals as they age, as they need more services than they did when they were younger. This is also true for people who are disabled or who have other special needs.

Table 1.1

Table 1.1 shows the number of students who took the test in each of the years 2002 through 2008. The number of students who took the test in each year is given in the table below.

| | 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 |
|------|---|---|---|---|----|----|----|----|----|----|
| 2 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 16 | | | | | | | | | | |
| 32 | | | | | | | | | | |
| 64 | | | | | | | | | | |
| 128 | | | | | | | | | | |
| 256 | | | | | | | | | | |
| 512 | | | | | | | | | | |
| 1024 | | | | | | | | | | |
| 2048 | | | | | | | | | | |

This chart provides many opportunities for students to recognize patterns and make discoveries which are related to rich areas in mathematics.

A teacher might start out by listing the powers of 2 down the chalk board in order and ask for students to continue giving entries according to the pattern the teacher has in mind. A similar activity can be performed for the primes across the top.

At this point we will diverge briefly from the major topic at hand to record an activity, brought out in our discussions, which relates to prime numbers.

The children are given some type of counters--pennies, pebbles, checkers, chips, pegs, etc.--and asked to see how many different rectangular arrays they can form with a given number of counters. Any n by m array is considered to be the same as any other n by m array as well as the same as any m by n array. Each child should conduct his own experiments and summarize his findings in a table like this one:

| Number of Tokens | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | ... |
|------------------------------|---|---|---|---|---|---|---|---|---|----|----|-----|
| Number of Rectangular Arrays | 1 | 2 | 2 | 3 | 2 | 1 | 2 | 2 | 2 | 2 | 1 | ... |

Students may now be told that those numbers for which there is only one rectangular array are the prime numbers (except for the number one which is or isn't depending upon the definition of particular author). This activity could be carried on along with getting kids to discover the pattern across the top of the chart. Upon looking at the table generated from investigating rectangular arrays, a student might be asked questions similar to the following.

Is there a number for which more than two rectangular arrays may be formed?

How many rectangles can be formed with six tokens? How many factors has six?

How many rectangles can be formed with nine tokens? How many factors has nine?

Find a number between 50 and 100 which has exactly 3 factors. How many such numbers are there?

The activity of forming rectangles can also be lowered down into the primary grades to teach multiplication. cf. The Advanced Montessori Method Vol. II by Maria Montessori.

In filling the powers of two-prime chart, column by column students recognize many patterns and are rewarded by this recognition since following the patterns greatly speeds their task. However, soon even the patterns become difficult to construct. This motivates students to make further discoveries which are immediately rewarding.

A very satisfying discovery is to abandon the set rules to divide a prime into the various powers of 2 and merely begin to double the previous remainder and record the excess above any multiple of the prime in question. This serves as a good introduction or application to modular arithmetic as well as some of the basic theorems on congruence such as Fermat's Little Theorem:

When p is prime and a is not divisible by p

$$a^{p-1} \equiv 1 \pmod{p}$$

H. Wills

A PROBLEM IN DIVIDING FRACTIONS

Take the first five prime numbers -- 2, 3, 5, 7, 11 -- and write them with four division signs of unequal length in the following way:

$$\frac{\frac{2}{\frac{3}{\frac{5}{\frac{7}{11}}}}}{}$$

The value of this expression is given by the rules that (a) the longest division sign is applied first, and (b) this rule is repeated until a simple fraction is obtained. Successively, one gets

$$2 \div \frac{\frac{3}{\frac{5}{\frac{7}{11}}}}{11} = 2 \times \frac{7}{11} \div \frac{3}{5} = \frac{2 \times 5 \times 7}{3 \times 11}$$

The final result need not be multiplied out.

By putting the division signs in a different sequence, a different value may be obtained. Thus,

$$\frac{\frac{\frac{2}{\frac{3}{\frac{5}{\frac{7}{11}}}}}{11}}{11} = 2 \div \frac{\frac{3}{\frac{5}{\frac{7}{11}}}}{11} = 2 \times \frac{5}{7} \div 3$$

$$= 2 \times 5 \div \frac{7}{11} \div 3 = \frac{2 \times 5 \times 11}{3 \times 7}$$

How many different arrangements of signs are possible? Do they all give different answers?

Try

$$\frac{\frac{2}{\frac{3}{\frac{5}{\frac{7}{11}}}}}{11}$$

How many distinct values are possible?

15

The primes are used so that all repetitions occur only as a result of equivalent arrangements of signs. The problem provides examples of non-associativity in arithmetic operations, gives practice in handling division by multiplying reciprocals, tests accuracy in handling compound fractions, and is challenging from the combinatorial point of view. For this latter aspect, have the students look at the same problem with 2, 3, or 4 division signs and try to find the rule for the number of distinct values obtained in the general case.

W. Jacobs

LOGIC PUZZLES

"I don't like spinach and I'm glad I don't
because if I did, I'd eat it and I hate it."

Logic puzzles can be useful in overcoming the purely linguistic difficulty of expressing things accurately in the ambiguous, redundant, variable medium of English. The syllogism puzzles of Lewis Carroll are especially good for this because the absurdity of their content serves to emphasize that only the form is of any importance in logic. These puzzles can be found, arranged in a graded sequence, in "Logical Nonsense", Putnam, 1934, pp. 505-546. For use with modern children, they need to be sifted and modified somewhat. In particular it might be a good idea to start with equivalent forms of single statements (which Lewis Carroll doesn't have) before going on to multi-statement puzzles. Following are a few examples (variants of L. C. assertions). For teaching purposes one would want many dozens of these to be worked first by common sense then by more systematic methods.

- A. Equivalent forms of single statements. Do the two statements say the same thing or different things? (Answers below)
1. No large birds live on honey. If a bird lives on honey, it is not large.
 2. No one, who forgets a promise, fails to do mischief. Anyone who forgets a promise does mischief.
 3. All, who are anxious to learn, work hard. All, who work hard, are anxious to learn.
 4. Prudent travellers carry plenty of small change.
Travellers without small change are imprudent.
 5. No child is healthy who takes no exercise.
Children who exercise are healthy.
 6. Some elderly ladies are talkative
Some talkative persons are elderly ladies.
 7. Nobody, who really appreciates Beethoven, fails to keep silent when the Moonlight Sonata is being played.
People, who keep silent when the Moonlight Sonata is being played, really appreciate Beethoven.

8. No drug is useful in a toothache, unless it relieves pain.
A drug that relieves pain is useful in a toothache.
9. None but a hop-scotch player knows real happiness.
All who know real happiness are hop-scotch players.
10. Whenever I do not take my umbrella it rains.
Whenever it does not rain, I take my umbrella.

Answers:

- | | |
|--------------|--------------|
| 1. same | 6. same |
| 2. same | 7. different |
| 3. different | 8. different |
| 4. same | 9. same |
| 5. different | 10. same |

After some practice with these, one can go on to puzzles involving two or more statements such as these:

B. Decide whether the conclusion follows from the two statements. (Answer below)

11. "I saw it in a newspaper."
"All newspapers lie."

Conclusion: It was a lie.

12. Every eagle can fly.
Some pigs cannot fly.

Conclusion: Some pigs are not eagles.

13. All, who are anxious to learn, work hard.
Some of these boys work hard.

Conclusion: Some of these boys are anxious to learn.

C. In each of the following examples, draw a conclusion, if possible.

14. All ducks waddle
Nothing that waddles is graceful.

15. Some unkind remarks are annoying.
No critical remarks are kind.

16. Canaries, that do not sing loud, are unhappy.
No well-fed canary fails to sing loud.

17. All puddings are nice.
 This dish is a pudding.
 No nice things are wholesome.
18. Nobody who really appreciates Beethoven fails to keep silent while the Moonlight Sonata is being played.
 Guinea-pigs are hopelessly ignorant of music.
 No one who is hopelessly ignorant of music ever keeps silent while the Moonlight Sonata is being played.

Answers:

11. Conclusion does not follow.
 ("All newspapers lie" is taken to mean "All newspapers sometimes lie." It could conceivably mean "All newspapers always lie." Such ambiguities should be avoided at first, later discussed.)
12. Conclusion follows.
13. Conclusion does not follow.
14. All ducks are ungraceful.
15. No conclusion.
16. Some ill-fed canaries are unhappy. (Lewis Carroll does not admit the possibility that the set of soft-singing canaries is empty. By modern standards one should admit this possibility. The important thing, though, is to be clear about the rules of the game. One implicit rule is that "some" means "one or more." Another implicit rule is that everything has at most two values: Canaries are either well-fed or ill-fed. Indifferently fed canaries do not exist.)
17. This dish is not wholesome.
18. Guinea-pigs do not really appreciate Beethoven.

These are a few examples to illustrate the tone and difficulty of these puzzles. For teaching purposes one should have dozens of them. After doing a great many by common sense one could introduce the set interpretation together with the usual visual aids (Venn diagrams). This should give the pupils quite a sense of power and enable them to go on to harder puzzles involving many statements. Several such are given in Logical Nonsense (loc. cit) including one with twenty statements. Other multi-statement logic puzzles can be found in the standard puzzle books, but without the Carrollian whimsy they often seem stodgy and artificial.

Because of its importance in mathematics, one should also practice a lot with the if then form. In particular the children should learn the work-horse rules about converses and negations.

Examples: In each set which statements are equivalent?

1. No child is healthy who takes no exercise.
If a child takes exercise, then he is healthy.
If a child is not healthy, then he takes no exercise.
2. No country, that has been explored, is infested with dragons.
If a country is infested with dragons, then it has not been explored.
If a country has not been explored then it is infested with dragons.
If a country has been explored, then it is not infested with dragons.

One can, of course, go on to more formal logic systems, Boolean Algebra, switching circuits, etc., but this is really a different subject, mathematics rather than linguistics. Puzzles of the Lewis Carroll kind are primarily linguistic: they concern equivalences between forms of English sentences. Skill in handling these equivalences is essential if one is to talk about mathematics in the English language.

Following is another logic puzzle that I happen to know and like.

The District Attorney

This is basically a puzzle, but it can be made into a guessing game by having someone in the know act out the part of the D.A. Even when people see the D.A. in action they often have difficulty divining his strategy, which can be made more mysterious by using different but logically equivalent questions each time through.

One of three suspects is guilty. The innocent ones can be counted on to tell the truth, but the guilty one may or may not tell the truth. The D.A. is to find out who is guilty with just two yes-no questions. At first glance it may seem impossible to get any information at all, but the D.A. does it, e.g., as follows:

D.A.: (to suspect A) All right A, either you or B did it, right?

A: No

D.A.: Then I can only conclude that C did it. (to suspect B): Will you confirm that C did it?

B: Yes

D.A.: Of course. C did it.

Alternative scenario:

D.A.: (to C) You look honest. Now tell me the truth. Did A do it?

C: No.

D.A.: (to A) If C were guilty he would have said, "yes" in order to frame you. Therefore it must be B. B did it, right?

A: No

D.A.: All right C. The jig is up. Honesty will get you nowhere.

C: But what if I had lied?

D.A.: I would have found out just the same. Dishonesty is no better than honesty.

C: You can't win.

D.A.: True.

False Proof by Induction

At the high school level after students have had experience with mathematical induction they should try this one:

Theorem: In any set of marbles all the marbles are the same color.

Proof (by induction) Let n be the number of marbles in the set. The theorem is certainly true if $n = 1$.

Induction: Suppose the theorem is true for n . Then it is surely true for $n + 1$, for if I remove any marble from a set of $n + 1$, the remaining ones, constituting a set of n , must all be the same color. Since this is true no matter which marble I remove, all $n + 1$ marbles must be the same color.

F. W. Sinden

DOES THE ORDER MAKE A DIFFERENCE?

1. In a single purchase, you are offered 3 successive discounts of 20 percent, 10 percent, and 5 percent, and can take them in any order that you wish. What order would you choose?
2. Arthur and Bob start a game with equal amounts of money. Arthur loses the first game and pays Bob 20 percent of his money. Then Bob loses the second game and pays Arthur 20 percent of the amount Bob has. Do they again have equal amounts of money?
3. Mr. Jones has two houses, which cost him the same amount. He sells one house at a 10 percent profit to Mr. Allen, who resells it to Mr. Baker at a 10 percent loss. Mr. Jones sells his second house at a 10 percent profit to Mr. Dahl. Which paid more, Mr. Baker or Mr. Dahl?

IDENTIFICATION PROBLEMS

The class of tricks based on what is generally called Gergone's Pile Problem provide an example of the power of numerical coding and an application of place system ideas. A typical version is the following: The "magician", A, asks B to deal 27 distinguishable cards in 3 equal piles, to select a card mentally and to announce which pile it is in. A then tells B to reassemble the cards by placing one pile on top of the other in any order. A notes the order in which the piles are assembled. After two repetitions of this cycle A tells B the position of his card in the reassembled deck.

If this trick is introduced to a fourth or fifth grade class in its 2 pile, 4 card form an analysis by the class should be possible and can be made to relate to binary numeration. The binary form and the general case can be investigated at an appropriate later time.

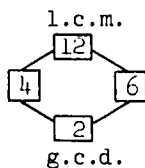
A complete discussion of several variations of this trick can be found in Martin Gardner's Mathematics, Magic, and Mystery (Dover, 1956).

W. Jacobs

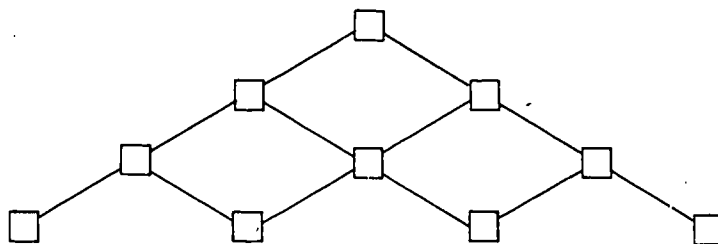
A GAME WITH FACTORS AND MULTIPLES (Grades 4-6)

The type of game outlined below depends solely on the multiplicative aspect of the positive integers. The objective is to explore factor relationships and incidentally to reinforce direct recall of the multiplication facts basic to computation.

The game is based on the g.c.d. and l.c.m. diagram exemplified by



To make a game consider a triangular array of say 10 boxes.



The game is played by 2, or perhaps 3, players, one of whom might be the teacher. Player A begins by filling in the top box. The other "players" in turn fill in the other boxes. In a simple version the only rule governing permissible entries is that the number put in the top box may never be used again.

The Play

After an entry is made any other player may challenge the player making it to show that the configuration is part of a "factor diagram". If the latter succeeds he wins (or scores so many points). If he fails he loses (or his challenger scores so many points). At any time a player believes his play makes the configuration part of a unique factor diagram he wins (or scores) upon convincing the other players of his assertion.

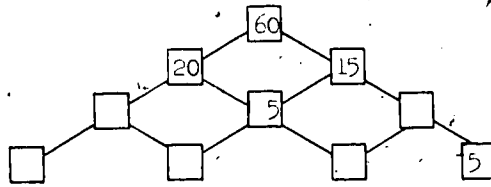
Here is a play.

(4)

(3)

(2)

(1)



| | | | | | |
|---|-------|------|-------|------|-------|
| A | 60(4) | | 20(3) | | 15(3) |
| B | | 5(1) | | 5(2) | |

This last play can be challenged.

Additional rules for admissible entries may be added. It may be required:
for example, that

- i) The entries in row (1) must be relatively prime in pairs, or
- ii) The entries in row (1) must be primes or 1, or
- iii) The number 1 may not be used.

W. Lister

A STOCHASTIC NUMBER LINE GAME

There are many commercial games which are essentially stochastic number line games. Typically, in these games the players move playing pieces along a track on the playing board, the length of the move being determined by a throw of a die or a pair of dice. The interest in such games is usually supplied by the system of rewards and penalties for stopping at specific positions along the path. While these games give interest children - and sometimes adults - the games are not particularly instructive since the play is purely a chance phenomenon, and the understanding of the probabilistic principles underlying the play is left to the perceptive player. In order to make the game instructive, particularly with regard to probability, it should be possible for the perceptive player to use one of the principles of probability to improve the chances of winning the play. We give an example of such a game.

the first will consist of seven on the number line although any discrete number of players represented will do. Any number of players may play and the game is played as follows. The players in turn throw a pair of dice and the other players measure a distance equal to the number of dots shown on the dice. It is then possible for a player to choose either a forward or backward direction for the move, that enables a player to improve his position on the line. The first player of his turn will throw the dice, and the other players will measure the distance, and then will proceed to 10. The player who is first to reach 10, this does not count and he must start again. The first player to reach a given number is the winner.

[illegible]

the position. The first step is to determine the best strategy is when simpler, the more complex the position. If the position is in the direction of the position, the position is simpler, the more complex the position. To analyze the best strategy, the position is simpler, the more complex the position. If the position is in its face, or if

it has one 1, two 2's, and three 3's.

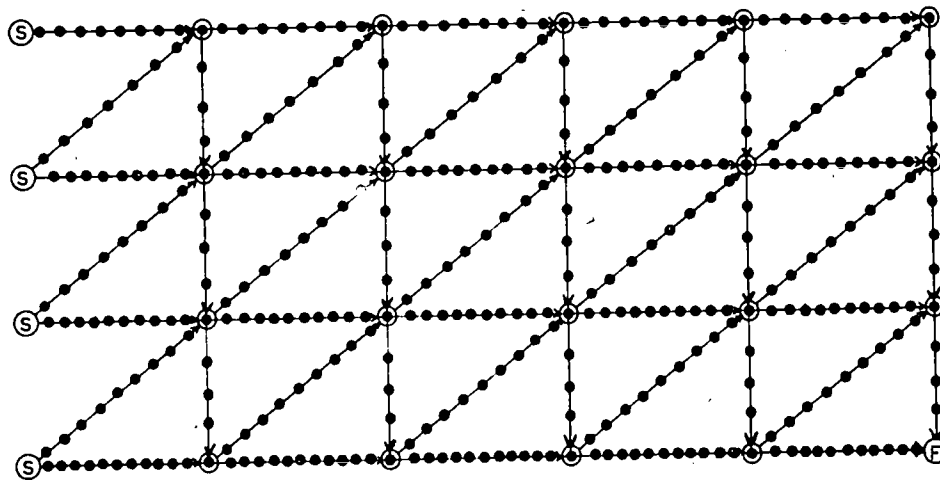
It is clear that games of this type give students considerable incentive to learn and use probability principles which will improve their chances of winning. With suitable choices of the basic distribution much of basic probability can be learned in this way. Furthermore, the kind of probabilistic reasoning required is very similar to the kind needed to make effective probability judgments in ordinary life situations.

R. P. Dilworth

GAMES ON DIRECTED GRAPHS

Stochastic games on the number line are a special case of stochastic games on directed graphs. Again there exist a number of commercial games which have the players move through a network, each move being determined by the throw of a die or a pair of dice. These games are likewise not particularly instructive since the play is usually a purely chance affair. However, games on directed graphs can easily be constructed in which an understanding of the underlying probability can be used to improve the strategy of play. Such games are potentially instructive particularly with regard to basic principles of probability. We give an example of a comparatively simple stochastic game on directed graph.

The playing board has a diagram as follows:



The game is played by four players who start in the four circular positions labeled S. They may draw lots to determine who starts in each position. They play in the order I, II, III, IV and, in his turn, each player throws a pair of dice to determine how far he will move along the directed graph. He may move along any path consistent with the arrows. His objective is to reach the finish position F in the fewest number of moves. The player first passing through a junction secures it for himself and other players may not follow paths passing through that junction.

The first player reaching F makes a score of $60 - n$ where n is the number of dots he has traversed in reaching the finish. All other players score zero. Note that the first player to reach F may still get a negative score.

This game clearly has a great many more strategic possibilities than the previous number line game. For example, player IV gains nothing by heading to the finish position directly since his score is then zero, but by taking a path higher into the graph he increases his chances of being cut off. Likewise, player I must decide whether he should take a path around the outside where he can make a good score but runs the risk of being cut off of taking a path into the graph and doing some cutting off himself. Perhaps the numbers of dots between junctions will have to be altered if a wide variety of strategies is to be obtained. Certainly, the distribution of the possible steps in each play must be used in estimating the effectiveness of a given strategy.

R. P. Dilworth

ELEMENTARY THEORY OF AREAS

There is a good deal of puzzle value in dissection problems and such experience should certainly help develop spatial geometric intuition especially the notions of Euclidian motions or congruences.

A pair of polygonal regions A and B are "equivalent" if A can be dissected into polygonal regions which can in turn be reassembled to form B. For example, in FIG. 1, A is equivalent to B, B to C and A to C.

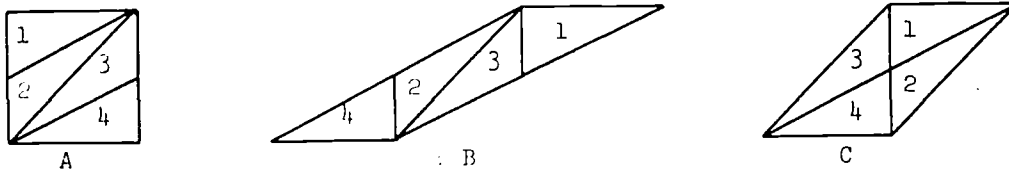


FIGURE 1

It is clear that any pair of equivalent polygonal regions have the same area; but the elegant thing is that the converse is also true: thus: "Every pair of polygonal regions with the same area are equivalent." It is this theorem which is illustrated here.

1. Show that the two parallelogram A and B, (see FIG. 2), which have the same heights and bases, are equivalent.

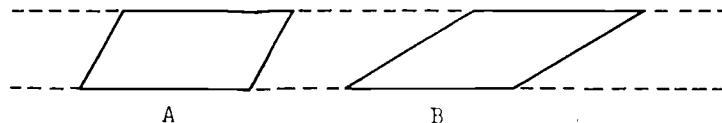


FIGURE 2

(Answer) superimpose bases



and a proper dissection becomes clearer:



2. Show that the two parallelograms A and B in FIG. 3 (in which they are illustrated with their bases superimposed) are equivalent.

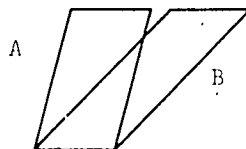
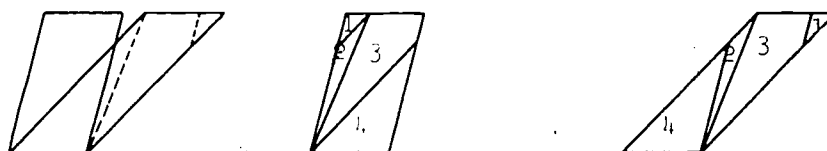
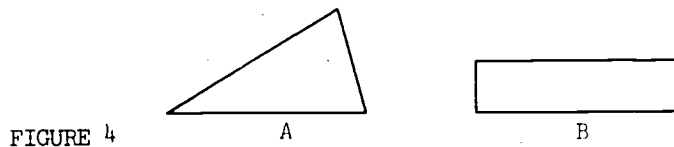


FIGURE 3

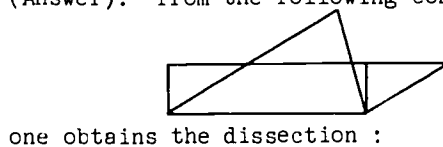
(Answer) This is similar to problem 1 but requires an extra step. The following illustration gives the idea:



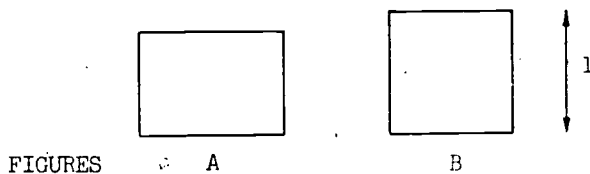
3. Show that the triangular region A is equivalent to the rectangular region B. (B has half the height of A).



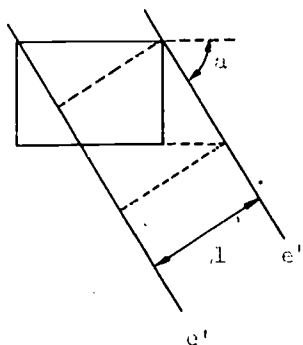
(Answer): from the following construction



4. Show that the rectangles A and B in FIG. 5 bound equivalent regions. The height of A is less than one and its length is greater than one. The height of B is equal to one and (since the areas bounded by A and B are equal) the length of B must be the common area of A and B.



(Answer): make use of following construction



choose angle α so that distance between e' and e'' is one.

5. Using the devices indicated in problems 1, 2, 3, 4 one can show that any triangle is equivalent to a rectangle of height one. It seems reasonable (and it can in fact be proved) that any polygon can be dissected into triangles; for example see FIG. 6.

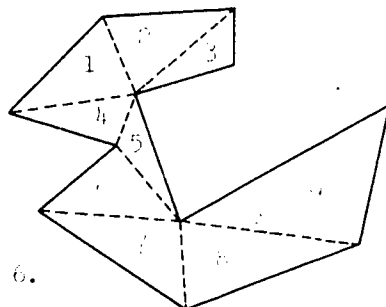
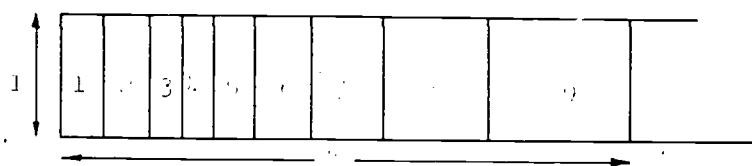


FIGURE 6.

Using the result on triangles one can dissect each of the triangles and rearrange into a rectangle of height one; thus



This not only computes the area "a" of the original polygonal region but also goes a long way toward showing the general result mentioned on page 1.

C. J. Titus

MAP COLORING PROBLEMS

In the following we are concerned with "maps"; for example,

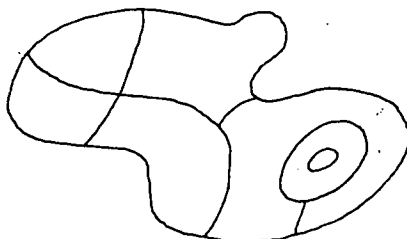


FIGURE 1;

and we are also concerned with the coloring of maps in a special way. We will say a map is "correctly colored" if no pair of bordering countries have the same color; for example, the following is a correct coloring" of the map in FIGURE 1:

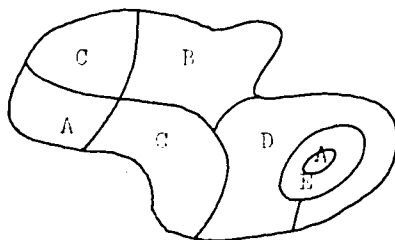
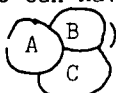


FIGURE 2;

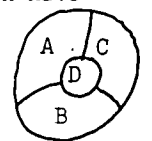
1. Can you "correctly color" the map in FIGURE 1 with fewer than the five colors used in FIGURE 2?
2. What is the least number of colors with which one can correctly color the map in FIGURE 1? (ans: 3)
3. Draw a map which can be correctly colored with two colors.
(sample answer: (A) (B) (C))

4. Draw a map that can be correctly colored with 3 colors but that cannot be correctly colored with 2 colors. (sample answer: the map in FIGURE 2).

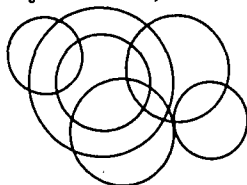
5. How many countries did you have in the map in problem 4? Can you achieve the same result with fewer countries? What is the smallest number of countries one can have in a map and still have the same result? (answer: 3; sample map



6. Draw a map that can be correctly colored with 4 colors but that cannot be correctly colored with 3 colors. Can you achieve the same result with fewer countries? What is the smallest number of countries one can have in a map and still have the same result? (answer: 4; sample map



7. Make a map using only circles; for example,



Correctly color your map with two colors. Notice that you cannot find such a "circle map" that requires more than 2 colors. Can you see why it is that every circle map can be colored with two colors?

(sample answer: write a number in each country which is the number of circles in which that country is contained. Color the even numbered countries one color and the odd numbered countries the other color.

C. J. Titus

A COMBINATORIAL PROBLEM SOLVED BY GEOMETRY

Consider the eight (2^3) triplets of 0's and 1's:

000
001
010
011
100
101
110
111

Can you arrange these in a cycle so that neighboring triplets differ in only one place? This problem arises in computer design where the 0's and 1's represent states of on-off devices (e.g. switches, lights, cores). A triplet of such devices is supposed to run through all of its eight states over and over again. For engineering reasons it may be awkward to switch two or more of the devices simultaneously. Therefore one would like to find a sequence that requires switching only one at each step.

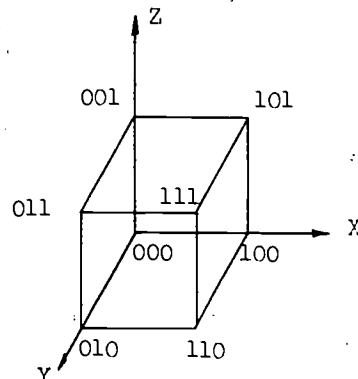
This looks like a hard problem. The number of possible cycles is enormous.

$$\frac{8!}{8} = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 5040$$

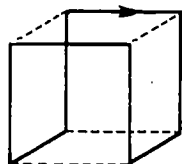
(Division by 8 because cyclic permutations indistinguishable)

It turns out, though, that a geometric interpretation makes the problem easy. Consider the triplets of 0's and 1's to be coordinates of points in 3 space.

The eight points represented by the triplets lie at the vertices of a cube.



Neighboring vertices (those joined by an edge) differ in exactly one coordinate. The problem then, is to find a closed path along the edges of the cube which passes through each vertex once. Such a path (Hamilton line) is easy to find:



Tracing out this path, one gets the solution:

000
001
101
100
110
111
011
010

All other Hamilton lines are rotations of the one shown above.

This problem offers an excellent opportunity to discuss higher dimensional geometry. To solve the problem with quadruples of 0's and 1's one considers Hamilton lines on a 4 - cube. To show what a 4 - cube is, one can display cubes of 0, 1, 2 and 3 dimensions, note the induction principle, use it to get the 4 - cube.

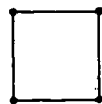
0 - cube

1 - cube



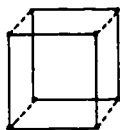
(2 0-cubes joined by a segment)

2 - cube



(2 1-cubes with corresponding vertices joined)

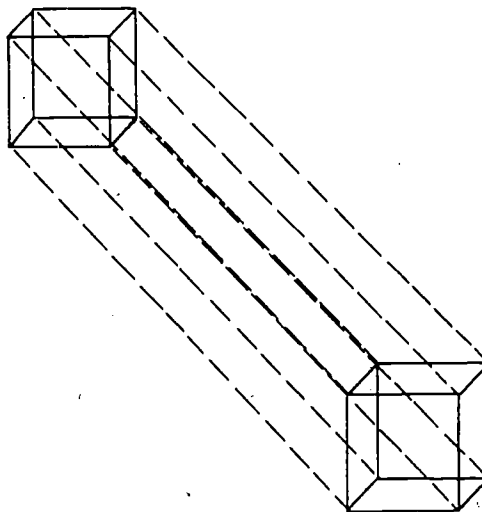
3 - cube



(2 2-cubes with corresponding vertices joined)

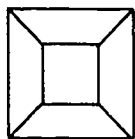
4 - cube

(2 3-cubes with corresponding vertices joined)



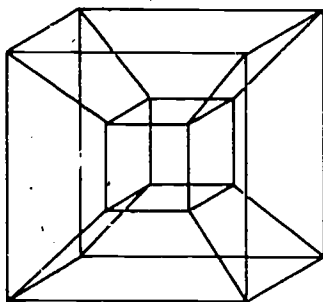
The drawing is neater if you put one cube inside the other:

3 - cube:



(one 2-cube inside the other)

4 - cube:



(one 3 - cube inside the other)

The construction of the Hamilton line can be generalized. Observe that the Hamilton line on a 3-cube first traverses one of the 2-cubes, then jumps to the other 2-cube, traverses it, jumps back to starting point. On the 4-cube: First traverse a 3-cube, jump to the other 3-cube, traverse it, jump back to starting point.

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